**MA (q) Process Basic Concepts**

A**q-order** **moving average process**, denoted **MA(q)** takes the form



Thinking of the subscripts i as representing time, we see that the value of y at time i+1 is a linear function of past errors. We assume that the error terms are independently distributed with a normal distribution with mean zero and a constant variance σ2. Thus



**Observation**: An MA(q) process can be expressed as

 

where zi = yi – μ. Thus, we can often simplify our analyses by restricting ourselves to the case where the mean is zero.

Using the lag operator, we can express a zero mean MA(q) process as



Where

 ,

defines a linear combination in the shift operator *Lq εi = εi-q*

**Property 1**: The mean of an MA(q) process is μ.

Proof:



**Property 2**: The variance of an MA(q) process is



Proof:



**Property 3**: The autocorrelation function of an MA(1) process is



Proof:



When h = 1



since E[εi-1] = 0.

When h > 1



Thus for h = 1, by Property 2



and for h > 1



**Property 4**: The autocorrelation function of an MA(2) process is



Proof:





Thus, when h = 1



and when h = 2



and when h > 2



It now follows that for h = 1, by Property 2

 [](http://www.real-statistics.com/wp-content/uploads/2019/01/image339.png)

and for h = 2



and for h > 2



**Calculating MA Coefficients using ACF**

If we know (or assume) that a time series can be fit by an MA(q) process, then we need to figure out the value of the parameters *μ*, *σ*2, *q*, *θ*1, …, *θq*. The initial approach to determining the value for *q* is to look at the ACF values for the time series under consideration. Since we know that for an MA(*q*) process, *ρk* = 0 for all *k > q*, we seek the first value for *q* where ACF(*q*) is approximately zero.

We next turn our attention to finding the other parameters that provide the best fit for the data. We start by looking at an MA(1) process

**yi = μ + εi + θ1εi-1 .**

We know that

Property 1: The mean is μ.

Property 2: The variance is



Property 3: The autocorrelation function is



We start by using the mean of the time series as μ. We then subtract this value from all the time series values to get a zero mean time series. We then calculate the variance s2 and r = ACF(1) of the time series. We can solve for θ1 using the equation



which is equivalent to the quadratic equation



which has the solutions



Actually θ1  above is really the estimated value of θ1 which typically has a hat over it. These solutions are real provided |r| < .5. It turns out that for large values of n



where



Also note that

