**MA (q) Process Basic Concepts**

A**q-order** **moving average process**, denoted **MA(q)** takes the form

[MA(q) process](https://i0.wp.com/www.real-statistics.com/wp-content/uploads/2016/04/image136z.png)

Thinking of the subscripts i as representing time, we see that the value of y at time i+1 is a linear function of past errors. We assume that the error terms are independently distributed with a normal distribution with mean zero and a constant variance σ2. Thus

[image137z](https://i2.wp.com/www.real-statistics.com/wp-content/uploads/2016/04/image137z.png)

**Observation**: An MA(q) process can be expressed as

[image138z](https://i2.wp.com/www.real-statistics.com/wp-content/uploads/2016/04/image138z.png)

where zi = yi – μ. Thus, we can often simplify our analyses by restricting ourselves to the case where the mean is zero.

Using the lag operator, we can express a zero mean MA(q) process as

[image139z](https://i1.wp.com/www.real-statistics.com/wp-content/uploads/2016/04/image139z.png)

Where

[image140z](https://i2.wp.com/www.real-statistics.com/wp-content/uploads/2016/04/image140z.png),

defines a linear combination in the shift operator *Lq εi = εi-q*

**Property 1**: The mean of an MA(q) process is μ.

Proof:

[https://i0.wp.com/www.real-statistics.com/wp-content/uploads/2019/01/image326.png?resize=503%2C22](https://i0.wp.com/www.real-statistics.com/wp-content/uploads/2019/01/image326.png)

**Property 2**: The variance of an MA(q) process is

[image141z](https://i2.wp.com/www.real-statistics.com/wp-content/uploads/2016/04/image141z.png)

Proof:

[https://i1.wp.com/www.real-statistics.com/wp-content/uploads/2019/01/image327.png?resize=602%2C44](https://i1.wp.com/www.real-statistics.com/wp-content/uploads/2019/01/image327.png)

**Property 3**: The autocorrelation function of an MA(1) process is

[image142z](https://i1.wp.com/www.real-statistics.com/wp-content/uploads/2016/04/image142z.png)

Proof:

[https://i2.wp.com/www.real-statistics.com/wp-content/uploads/2019/01/image328.png?resize=464%2C20](https://i2.wp.com/www.real-statistics.com/wp-content/uploads/2019/01/image328.png)[https://i2.wp.com/www.real-statistics.com/wp-content/uploads/2019/01/image329.png?resize=395%2C20](https://i2.wp.com/www.real-statistics.com/wp-content/uploads/2019/01/image329.png)

When h = 1

[https://i0.wp.com/www.real-statistics.com/wp-content/uploads/2019/01/image330.png?resize=233%2C20](https://i0.wp.com/www.real-statistics.com/wp-content/uploads/2019/01/image330.png)

since E[εi-1] = 0.

When h > 1

[https://i1.wp.com/www.real-statistics.com/wp-content/uploads/2019/01/image331.png?resize=107%2C20](https://i1.wp.com/www.real-statistics.com/wp-content/uploads/2019/01/image331.png)

Thus for h = 1, by Property 2

[https://i1.wp.com/www.real-statistics.com/wp-content/uploads/2019/01/image332.png?resize=263%2C42](https://i1.wp.com/www.real-statistics.com/wp-content/uploads/2019/01/image332.png)

and for h > 1

[https://i1.wp.com/www.real-statistics.com/wp-content/uploads/2019/01/image333.png?resize=208%2C40](https://i1.wp.com/www.real-statistics.com/wp-content/uploads/2019/01/image333.png)

**Property 4**: The autocorrelation function of an MA(2) process is

[https://i1.wp.com/www.real-statistics.com/wp-content/uploads/2016/04/image143z.png?resize=384%2C41](https://i1.wp.com/www.real-statistics.com/wp-content/uploads/2016/04/image143z.png)

Proof:

[https://i1.wp.com/www.real-statistics.com/wp-content/uploads/2019/01/image334.png?resize=592%2C20](https://i1.wp.com/www.real-statistics.com/wp-content/uploads/2019/01/image334.png)[https://i0.wp.com/www.real-statistics.com/wp-content/uploads/2019/01/image335.png?resize=498%2C20](https://i0.wp.com/www.real-statistics.com/wp-content/uploads/2019/01/image335.png)

[https://i2.wp.com/www.real-statistics.com/wp-content/uploads/2019/01/image336.png?resize=476%2C20](https://i2.wp.com/www.real-statistics.com/wp-content/uploads/2019/01/image336.png)

Thus, when h = 1

[https://i0.wp.com/www.real-statistics.com/wp-content/uploads/2019/01/image337.png?resize=412%2C20](https://i0.wp.com/www.real-statistics.com/wp-content/uploads/2019/01/image337.png)

and when h = 2

[https://i1.wp.com/www.real-statistics.com/wp-content/uploads/2019/01/image338.png?resize=234%2C20](https://i1.wp.com/www.real-statistics.com/wp-content/uploads/2019/01/image338.png)

and when h > 2

[https://i1.wp.com/www.real-statistics.com/wp-content/uploads/2019/01/image331.png?resize=107%2C20](https://i1.wp.com/www.real-statistics.com/wp-content/uploads/2019/01/image331.png)

It now follows that for h = 1, by Property 2

[[https://i2.wp.com/www.real-statistics.com/wp-content/uploads/2019/01/image339.png?resize=329%2C42](http://www.real-statistics.com/wp-content/uploads/2019/01/image339.png)](http://www.real-statistics.com/wp-content/uploads/2019/01/image339.png)

and for h = 2

[https://i1.wp.com/www.real-statistics.com/wp-content/uploads/2019/01/image340.png?resize=311%2C42](https://i1.wp.com/www.real-statistics.com/wp-content/uploads/2019/01/image340.png)

and for h > 2

[https://i1.wp.com/www.real-statistics.com/wp-content/uploads/2019/01/image333.png?resize=208%2C40](https://i1.wp.com/www.real-statistics.com/wp-content/uploads/2019/01/image333.png)

**Calculating MA Coefficients using ACF**

If we know (or assume) that a time series can be fit by an MA(q) process, then we need to figure out the value of the parameters *μ*, *σ*2, *q*, *θ*1, …, *θq*. The initial approach to determining the value for *q* is to look at the ACF values for the time series under consideration. Since we know that for an MA(*q*) process, *ρk* = 0 for all *k > q*, we seek the first value for *q* where ACF(*q*) is approximately zero.

We next turn our attention to finding the other parameters that provide the best fit for the data. We start by looking at an MA(1) process

**yi = μ + εi + θ1εi-1 .**

We know that

Property 1: The mean is μ.

Property 2: The variance is

[image016c](https://i2.wp.com/www.real-statistics.com/wp-content/uploads/2016/04/image016c.png)

Property 3: The autocorrelation function is

[image017c](https://i2.wp.com/www.real-statistics.com/wp-content/uploads/2016/04/image017c.png)

We start by using the mean of the time series as μ. We then subtract this value from all the time series values to get a zero mean time series. We then calculate the variance s2 and r = ACF(1) of the time series. We can solve for θ1 using the equation

[image018c](https://i0.wp.com/www.real-statistics.com/wp-content/uploads/2016/04/image018c.png)

which is equivalent to the quadratic equation

[image019c](https://i1.wp.com/www.real-statistics.com/wp-content/uploads/2016/04/image019c.png)

which has the solutions

[image020c](https://i1.wp.com/www.real-statistics.com/wp-content/uploads/2016/04/image020c.png)

Actually θ1  above is really the estimated value of θ1 which typically has a hat over it. These solutions are real provided |r| < .5. It turns out that for large values of n

[image021c](https://i1.wp.com/www.real-statistics.com/wp-content/uploads/2016/04/image021c.png)

where

[image022c](https://i2.wp.com/www.real-statistics.com/wp-content/uploads/2016/04/image022c.png)

Also note that

[image023c](https://i1.wp.com/www.real-statistics.com/wp-content/uploads/2016/04/image023c-1.png)