**Autoregressive Processes Basic Concepts**

In a simple linear regression model, the predicted dependent variable is modeled as a linear function of the independent variable plus a random error term.



A **first-order** **autoregressive process**, denoted **AR(1)**, takes the form



Thinking of the subscripts i as representing time, we see that the value of y at time i+1 is a linear function of y at time i plus a fixed constant and a random error term. Similar to the ordinary linear regression model, we assume that the error terms are independently distributed based on a normal distribution with zero mean and a constant variance σ2 and that the error terms are independent of the y values. Thus





Similarly, a **second-order autoregressive process**, denoted **AR(2)**, takes the form



and a **p-order autoregressive process**, **AR(p)**, takes the form



**Property 1**: The mean of the yi in a stationary AR(p) process is



Proof:

Since the process is stationary, for any k, E[yi] = E[yi-k], a value which we will denote μ. Since E[εi] = 0,  E[φ0] = φ0 and



it follows that



Solving for μ yields the desired result.

**Property 2**: The variance of the yi in a stationary AR(1) process is



Proof:

Since the yi and εi are independent, by basic properties of variance, it follows that





Since the process is stationary, yi = yi-1, and so



Solving for var(yi) yields the desired result.

**Property 3**: The lag h autocorrelation in a stationary AR(1) process is



Proof:

First note that for any constant a, cov(a+x, a+y) = cov(x,y). Thus, cov(yi,yj) has the same value even if we assume that φ0 = 0, and similarly for var(yi) = cov(yi,y*i*). Thus, it suffices to prove the property when φ0 = 0. In this case, by Property 1, μ = 0, and so cov(yi,yj) = E[yiyj].

Thus



since by the stationary property, E[yi-1,y*i-k*] = γi-1. Now, by induction on k, it is easy to see that

 

Hence



**Property 4**: For any stationary AR(p) process. The autocovariance at lag k > 0 can be calculated as



Similarly the autocorrelation at lag k > 0 can be calculated as



Here we assume that γh = γ-h and ρh = ρ-h if h < 0, and ρ0 = 1.

These are known as the **Yule-Walker equations**.