Time Series Analysis

"A time series is a set of statistical observations arranged in chronological order". —*Morris Humburg*

Introduction

So far we have discussed different methods on different measurements which are all related to statistical analysis. But we have not yet considered regarding the effect of time. But in the study of economic problems the chronological variation plays a vital role in the study of supply and demand, the rise and fall of price of a commodity, etc. In geography, the study of atmospheric pressure, humidity, rainfall, etc., are mostly related with time. Studies which relate the analysis of a variable with a specific period of time (either long or short) come under the ambit of time series analysis. The statistical series of quantitative data when arranged chronologically is known as time series. The analytic study of the time series is important so as to forecast regarding the fluctuation of the data in future, on the basis of the trend studied from the data. So, time series analysis may be regarded as a decision making factor of any concern for their future plan and estimate.

Components of Time Series

Through the detailed study of the time series we can extract an idea about the changes in the effects of various factors which interact simultaneously. These effects are classified in some major categories. These categories are known as the components of time series. The components are :

1) Secular trend
2) Seasonal variations
3) Cyclical fluctuations
4) Irregular variations.

A brief discussion may be done regarding components for further clarification.

1. Secular trend

The word 'trend' means the tendency. So, secular trend is that component of the time series which gives the general tendency of the data for a long period. It is smooth, regular and long-term movement of a series. The steady growth of the sale status of a particular commodity of a company or the fall of demand for a certain article for long years can be studied through this secular trend. Growth of population in a locality for considerably long years is the good example of secular trend. Rapid fluctuation cannot give the trend.

2. Seasonal variation

If we observe the sale structure of clothes in the market we will find that the sale curve is not uniform throughout the year. It shows different trend in different seasons like summer, winter, etc., but the word 'season' is never strictly means the usual seasons in the year. In every locality the change may happen due to some particular well-known religious festivals, like Durga Puja, Id, X-mass, etc. It can also be seen that each and every year, sale structure is more or less same as the previous year in those periods. So, it is that component of a time series which occurs uniformly and regularly. This variation is periodic in nature and regular in character. In each and every year we find the price of agro-products comes down at the time of harvest. It is also one kind of seasonal variation.

3. Cyclical fluctuations

Apart from seasonal variation there is another type of fluctuations whose period is usually more than a year. But seasonal variation has a period of one year or less. This type of fluctuation is the effect of business cycle. In every business there are four important phases—(i) prosperity, (ii) decline, (iii) depression and (iv) improvement or regain. The time from prosperity to regain is a complete cycle. So, this cycle will never show regular periodicity. But which is important that the period of a cycle may differ but the sequence of changes is more or less regular, and this fact of regularity enables us to study cyclical fluctuations.

4. Irregular variations

Irregular variations are those which are quite unpredictable. The effects due to flood, draughts, famine, devastating storms, earthquake or any natural calamities, are known as irregular variations. The variations which includes other than trend, seasonal and cyclical variations are all irregular. Sometimes cyclical fluctuations may generate from those calamities, but at that time it is difficult to distinguish cyclical fluctuation and irregular variation.

Necessity

The time series analysis helps us to understand the past behaviour of a variable. Through this analysis we can easily isolate the factors which are responsible for the ups and downs. The executives of a concern may take future steps to promote the sales of commodity. Whether any technological upliftment is necessary or simply advertising section should be geared up, can be well adjudged through this analysis. Intra-year variations can also be chalked out. Appointment of skilled personnel in the production or sales sector can be minimised through the better understanding with circumstances.

Classical Model

In classical or traditional time series analysis, it is assumed that there is a multiplicative relationship among the factors.

If Y denotes the value of the variable at time t and T, C, S and I represent trend, cyclical fluctuation, seasonal variation and irregular variation respectively, then

$$Y = T \times C \times S \times I.$$

On the other hand, it is also assumed that

$$Y = T + C + S + I,$$

which is additive relationship among the components.

Though it should be kept in mind that all those four components may not be present at a time in each time series. Also what is important is that each factor may not be independent to each other.

Editing of Data

Before using the time series data for analysis to isolate and measure the four components, we must ensure for the comparability up to the requisite level. So, for editing the data we must take care of the following factors :

(i) Calendar variation

For monthly data, as each month is not of equal duration, it is better to convert the data day-wise. In that case each data should be divided by the number of days in that month for which that data is supplied. Again, that daily data may be converted to weekly data, simply multiplying the daily data by seven.

(ii) Change of population

In a market study, if a reputed company likes to survey the sales position of a commodity through a comparison with last 10 or 15 years' data, the correct analysis will not be revealed as with that long period the population has drastically changed due to new birth, death and migratory population. So, users of the commodity should be calculated per thousand in each year, in a particular locality. In that case total population can be estimated by taking the average growth into consideration. We may depend upon the census report for this purpose.

(iii) Change of price

For any sales-oriented data time series analysis will be somewhat erratic if we consider simply sales price only. From our experience, we observe that prices are changing most frequently. To avoid this effect we may consider the unit of commodities but not their face values. So, the total price should be converted to the units sold and in case of more than one commodity the price index should be considered.

(iv) Change of other factors

Apart from those factors discussed above, there may be some other factors due to which some meaningful analysis may not be possible. During the span of time series taxes in different sectors are changing. So, the analysis on the revenue collected by state or central government must be adjusted accordingly.

Measurement of Trend

We have already discussed that 'trend' is the tendency. The reasons for the measurement of trend is firstly to study the behaviour of the variable in the long run. This study is possible only when the effects of the other components of the time series are eliminated. Secondly, to study the regular or irregular variation, which is possible only when trend values are isolated.

The following methods can be adopted for the measurement of trend :

(i) Graphic method.

(ii) Semi-average method.

(iii) Moving average method.

(iv) Curve-fitting by method of least squares.

Graphic Method

This is simplest of all the methods, also it is free from mathematical calculations. Initially points are to be plotted on a graph paper, on horizontal axis year marks may be plotted and on the vertical axis data may be taken. Then simply by inspection a line is to be drawn so that the fluctuations on one direction become approximately equal to those on the other direction. This trend line should be drawn carefully so that the following conditions are fulfilled :

(1) It should be smooth.

(2) The sum of the vertical deviations from the trend line of the points above the line should be more or less equal to the sum of the vertical deviations of the points below the line.

(3) Sum of the squares of the vertical deviations of all points from the trend line should be minimum.

Example 15.1. Fit a trend line by freehand method from the following data :

The number of eggs sold by a small poultry merchant in a year month-wise

Jan.	480	Jul.	560
Feb.	356	Aug.	420
Mar.	520	Sep.	660
Apr.	460	Oct.	580
May.	420	Nov.	520
June.	360	Dec.	480

Solution :

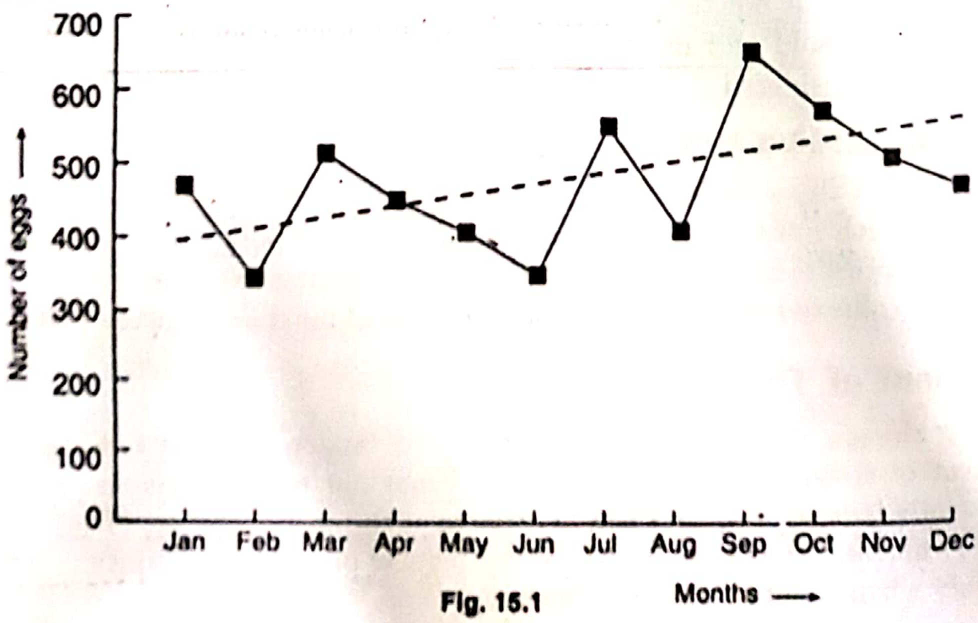

Fig. 15.1

Merits :

(1) This method is most simple and needs no mathematical calculations.

(2) For an experienced statistician, it is a time-saving method. He can easily set the line at a glance.

Limitations :

(1) For beginners it is difficult to draw the line with eye estimation.

(2) This is purely subjective as it depends on the judgement of the statistician.

(3) Further prediction may be far from the proper estimate as it is fully dependent on eye estimation.

(4) It requires a lot of experience to draw the line.

Method of Semi-averages

In this method we generally divide the given data in two equal parts. An average of each part is done. Now those two averages are plotted against the middle points of each span of time. Now those two points are joined by a straight line.

Problem 15.1. Fit a line through the method of semi-average of the following data :

The amount of electric bills of a company in the year 2002

Month	Amount in Rs. ('000)	Month	Amount in Rs. ('000)
Jan	3.57	Jul	4.12
Feb	2.89	Aug	3.93
Mar	3.14	Sep	3.05
Apr	1.58	Oct	2.88
May	2.76	Nov	3.17
Jun	3.42	Dec	3.46

Solution : As there are 12 data from January to December, we will divide the total span in two groups one from January to June and other from July to December.

Month	Amount in Rs. ('000)	Average in Rs. ('000)	Month	Amount in Rs. ('000)	Average in Rs. ('000)
Jan	3.57		Jul	4.12	
Feb	2.89		Aug	3.93	
Mar	3.14		Sep	3.05	
		2.89			3.43
Apr	1.58		Oct	2.88	
May	2.76		Nov	3.17	
Jun	3.42		Dec	3.46	

Here the points for data corresponding to each month are plotted in the usual way. But the points for average data are plotted against the mid-points of March and April, and September and October, i.e., against 15th of March and 15th of September.

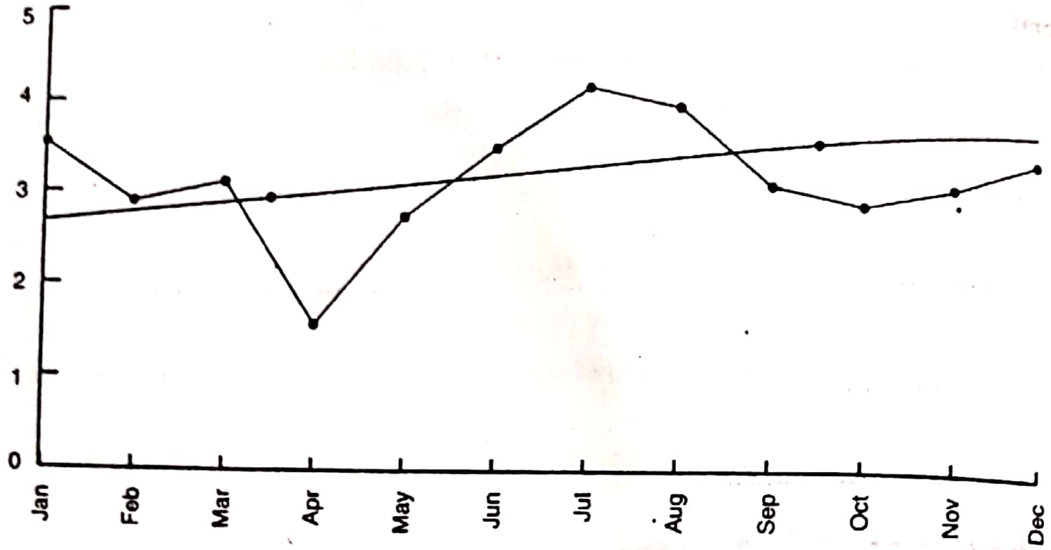

Fig. 15.2

We have discussed the method of fitting a line when the number of data given to be even. But in case when the number of data is odd, we are to form the groups by leaving the middle one.

Problem 15.2.

The amount of electric bills of a company in the year 2002

Month	**Amount in Rs. ('000)**	**Month**	**Amount in Rs. ('000)**
Jan	3.57	May	2.76
Feb	2.89	Jun	3.42
Mar	3.14	Jul	4.12
Apr	1.58		

Draw the trend line by the method of semi-average and estimate the bill for August 2002.

Solution : Here for trend line we are to plot Rs. 3,200 and Rs. 3,430 corresponding to the months of February and June respectively.

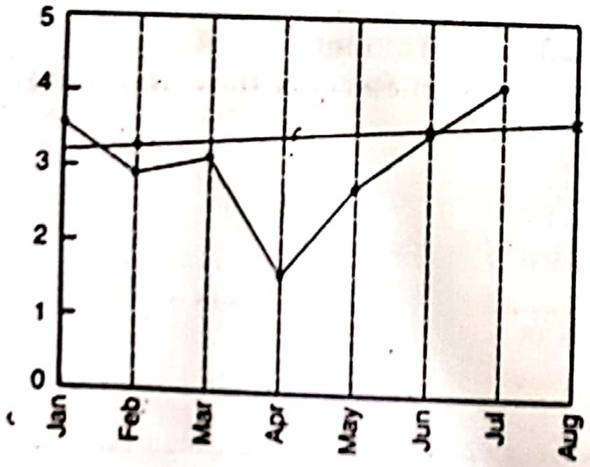

Fig. 15.3

Month	**Amount in Rs. ('000)**	**Average in Rs. ('000)**
Jan	3.57	
Feb	2.89	3.2
Mar	3.14	
Apr	1.58	
May	2.76	
Jun	3.42	3.43
Jul	4.12	

Hence from the above trend line the estimated amount of bill in August 2002 is Rs. 3545.

Merits :

(1) This method is easier than any other mathematical methods like moving average and least squares.

(2) This method is purely objective and systematic. The answer is rigidly framed.

Limitations :

(1) This method is suitable only when the trend is linear, otherwise it fails. In fact, when the trend is parabolic, the semi-average method is helpless to produce correct estimate.

(2) If there is any highly extreme values in either half, the trend line will yield considerable error.

(3) If the time span within the middle of each span is small, the estimated trend value becomes absurd.

Method of Moving Average

This method gives usually satisfactory result. Through this method fluctuations are reduced, so we get trend values with higher degree of accuracy. In this method, the average values for a number of spans (years or months or weeks or days), etc., are first calculated. Now this calculated average is plotted against the middle of total time for which the average is calculated. If the time span of the data be a year, generally, 3-yearly, 5-yearly or 7-yearly average is taken. It is wise that the period selected for the moving average must coincide with the length of the cycle. Through this technique we can avoid the effect of cyclical fluctuations. But in most of the economic data, cycles generally have different periods. In that case the average of all the periods may be taken as the period of moving average. But in that case cyclical fluctuation can be fully avoided. When the data is arranged chronologically in the form $a, b, c, d, e, f, \ldots$ by 3-yearly moving average we mean $\frac{a+b+c}{3}$, $\frac{b+c+d}{3}$, $\frac{c+d+e}{3}, \ldots$ and 5-yearly moving average we mean $\frac{a+b+c+d+e}{5}$, $\frac{b+c+d+e+f}{5}, \ldots$ and so on.

Problem 15.3. Calculate the 3-weekly moving average of the data on the number of weekly incoming calls received by an executive for successive 12 weeks :

342, 257, 512, 430, 292, 592
488, 324, 462, 548, 398, 298

Solution : **Calculation of 3-weekly moving average :**

Week	No. of calls	3-weekly total	3-weekly moving average
First	342	–	
Second	257	1111	370.3
Third	512	1199	399.7
Fourth	430	1234	411.3
Fifth	292	1314	438.0
Sixth	592	1372	457.3
Seventh	488	1404	468.0
Eighth	324	1274	424.7
Ninth	462	1334	444.7
Tenth	548	1408	469.3
Eleventh	398	1244	414.7
Twelfth	298	–	

Problem 15.4. Calculate 3-yearly and 5-yearly moving averages for the number of successful candidates in B.Sc.(Final Exam.) in a college and exhibit the trend graphically :

Year	No. of students	Year	No. of students
1981	3	1986	32
1982	7	1987	34
1983	15	1988	28
1984	22	1989	37
1985	18	1990	35

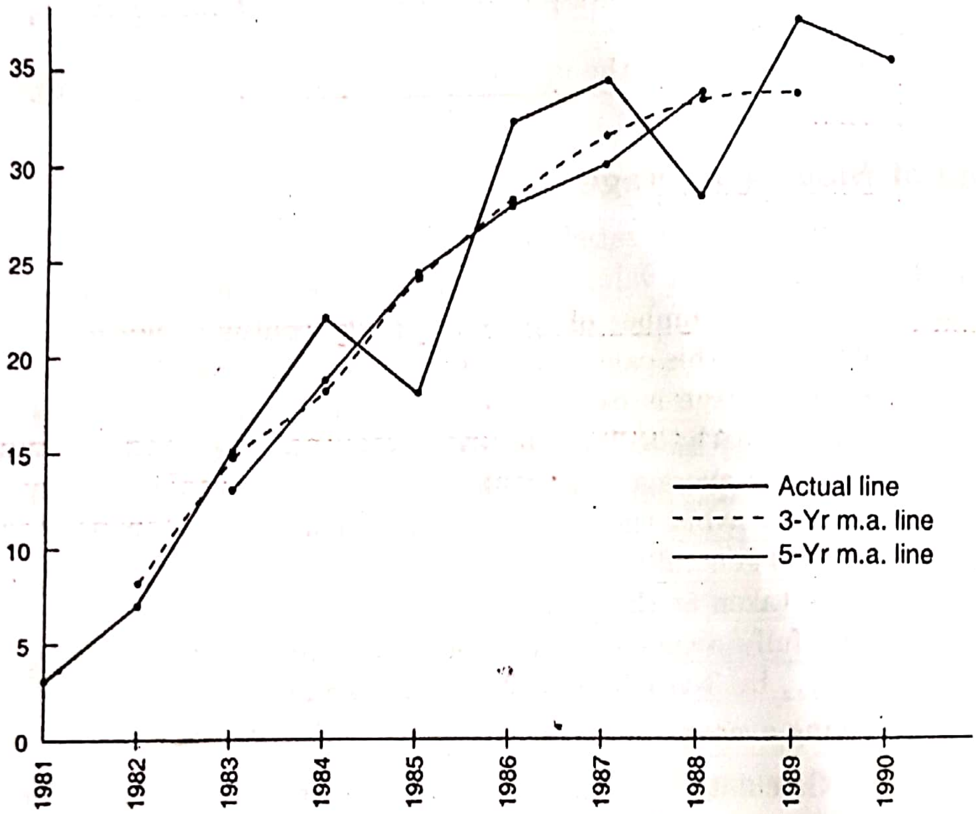

Fig. 15.4

Solution : Calculation for 3-yearly and 5-yearly moving average.

Year	No. of students passed	3-yearly moving total	3-yearly moving average	5-yearly moving total	5-yearly moving average
1981	3	–	–	–	–
1982	7	25	8.3	–	–
1983	15	44	14.7	65	13.0
1984	22	55	18.3	94	18.8
1985	18	72	24.0	121	24.2
1986	32	84	28.0	134	26.8
1987	34	94	31.3	149	29.8
1988	28	99	33.0	166	33.2
1989	37	100	33.3	–	–
1990	35	–	–	–	–

Even Period of Moving Average

When the period is even, the moving total and consequently moving average fall midway between two periods. Now, if we place the data in the middle of two periods, that will not coincide with original periods. To avoid this inconvenience we further take two-item moving average, so that the new moving average corresponds to original time period. This process is called 'centring'.

Problem 15.5. Calculate 4-yearly moving average for the production of a biscuit company given as under :

Year	1991	1992	1993	1994	1995	1996
Units ('000)	247	472	498	512	360	480
Year	1997	1998	1999	2000	2001	2002
Units ('000)	527	540	420	490	540	570

Draw the trend curve.

Solution : **Calculation of 4-yearly moving average**

Year	Production units ('000)	4-yearly moving total	4-yearly moving average	2-item moving total (centered)	4-yearly moving average (centered)
1991	247	–			
1992	472	–			
		1729	432.25		
1993	498			892.75	446.4
		1842	460.50		
1994	512			923.00	461.5
		1850	462.50		
1995	360			932.25	466.1
		1879	469.75		
1996	480			946.50	473.2
		1907	476.75		
1997	527			968.50	484.2
		1967	491.75		
1998	540			986.00	493.0
		1977	494.25		
1999	420			991.75	495.9
		1990	497.50		
2000	490			1002.50	501.2
		2020	505.00		
2001	540	–			
2002	570	–			

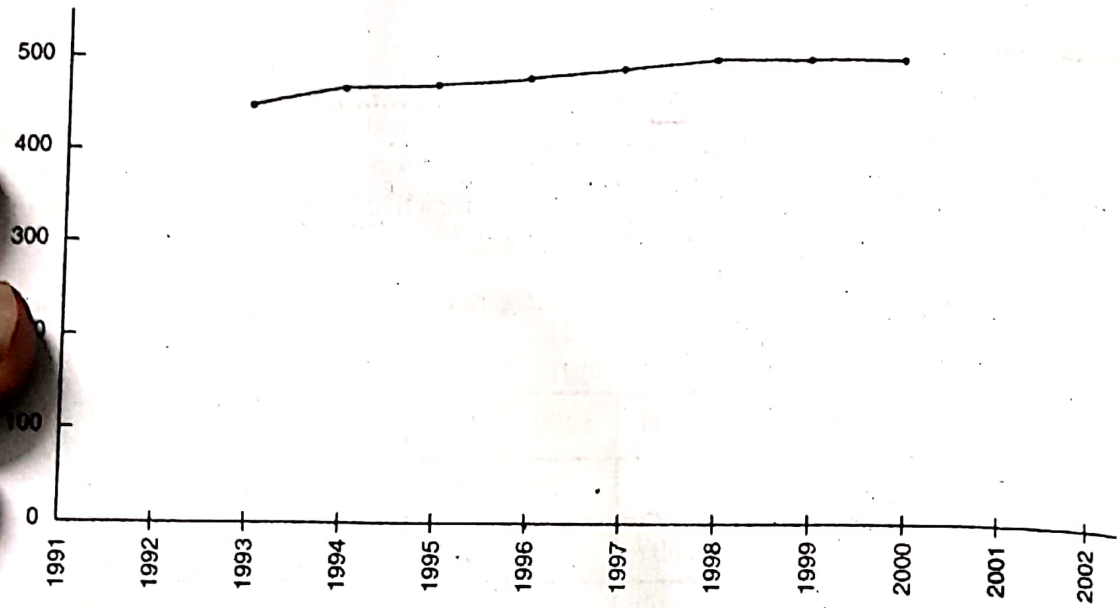

Fig. 15.5

Merits :

(1) This method is free from mathematical complicacy, and simple in comparison with the method of least squares.

(2) The most advantage of this method is that if some more data can be collected and added we can extend the trend line further, which is a very flexible nature of calculation.

(3) If we can adjust the period of moving average with the period of cyclical fluctuation, then the trend can be made free from fluctuation effect.

(4) As moving average follows the general movement of the data, so the advantage is that, it does not depend on the choice of individual.

Limitations :

(1) Through this method trend values cannot be obtained for the end points. The initial and final points are always left.

(2) The choice of period of moving average is flexible and there is no hard and fast rules for the selection of period, which is somewhat subjective.

(3) In case of non-linear trend, complicacy arises. For concave upward trend, the trend value is overestimated through moving average method and for convex upward trend, the trend value is underestimated.

(4) As the method of moving average assumes no law of change, i.e., not guided by mathematical function, it cannot be used for forecasting the future trend.

(5) As in most of the economic data, cyclical fluctuations are generally irregular, no moving average can remove the cycle completely.

Method of Least Squares

The most satisfactory method to determine the trend is the method of least squares. Through this method we obtain an objectively determined mathematical equation. In both cases for linear and non-linear trend we can use this method. The trend line obtained through this process is also known as **Line of best fit.**

If Y_c is the computed value of Y from the trend line, the following conditions must be satisfied:

(i) $\sum(Y - Y_c) = 0$ (ii) $\sum(Y - Y_c)^2$ is minimum.

i.e., the sum of vertical displacements of the points above the trend line is same as the sum of vertical displacements from the trend line of the points below the trend line and the sum of the squares of deviations of the original values and the corresponding computed values should be minimum.

(a) Linear Trend

If X represents the time (year, month or day or any span of time) and Y represents the value of data, then for a linear trend, we can adjust constants a and b such that

$$Y_c = a + bX \quad (1)$$

then taking summation for n such values, we have,

$$\sum Y = na + b\sum X. \quad (2)$$

Again multiplying (1) by X and taking summation, we get

$$\sum XY = a\sum X + b\sum X^2. \quad (3)$$

Solving (1) and (3) we can easily get the values of a and b.

Problem 15.6. Fit a straight line trend equation by the method of least squares from the following data and then estimate the trend value for the year 1985 :

Year :	1971	1972	1973	1974	1975	1976	1977	1978	1979	1980
Value ('000) :	65	80	84	75	77	71	76	74	70	68

[CUCH 1982]

Solution : Here the number of years is even. So, we may take the middle of 1975–76 as the origin, which will make $\sum x = 0$. So, the normal equations will be reduced to the form,

$$\sum y = na \quad \text{and} \quad \sum xy = b\sum x^2.$$

So, $$a = \frac{1}{n}\sum y \quad \text{and} \quad b = \frac{\sum xy}{\sum x^2}.$$

Calculation for trend line

Year	Value (y)	u	x	xy	x^2
1971	65	$\frac{-9}{2}$	−9	−585	81
1972	80	$\frac{-7}{2}$	−7	−560	49
1973	84	$\frac{-5}{2}$	−5	−420	25
1974	75	$\frac{-3}{2}$	−3	−225	9
1975	77	$\frac{-1}{2}$	−1	−77	1

Contd.

Year	Value (y)	u	x	xy	x^2
1976	71	$\frac{1}{2}$	1	71	1
1977	76	$\frac{3}{2}$	3	228	9
1978	74	$\frac{5}{2}$	5	370	25
1979	79	$\frac{7}{2}$	7	490	49
1980	68	$\frac{9}{2}$	9	612	81
	740		0	−96	330

Using the values in the normal equations, we get

$$a = \frac{1}{n}\sum y = \frac{740}{10} = 74.$$

$$b = \frac{\sum xy}{\sum x^2} = -\frac{96}{330} = -0.291.$$

So, the trend equation is

$$y = 74 - 0.291x.$$

Now for the year 1985, $x = 19$, so the trend value in 1985 is

$$y_{1985} = 74 - 0.291 \times 19 = 68.471.$$

So, the estimated trend value in the year 1985 $= 68.471 \times 1000 = 68471$.

Example 15.7. Fit a straight line trend equation, using least squares method for the following data on the price per kilogram of Rohu fish (within 1 kg wt.) in local market. Find also the trend rate in each year :

Year :	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000
Rate per kg (Rs.) :	16	22	24	20	32	40	44	48	50	52	60

Solution : **Calculation for trend values**

Year	Rate/kg in Rs. (y)	x	xy	x^2	Trend values in Rs.
1990	16	−5	−80	25	15.00
1991	22	−4	−88	16	19.42
1992	24	−3	−72	9	23.84
1993	20	−2	−40	4	28.26
1994	32	−1	−32	1	32.67
1995	40	0	0	0	37.10
1996	44	1	44	1	41.51
1997	48	2	96	4	45.93
1998	50	3	150	9	51.17
1999	52	4	208	16	54.76
2000	60	5	300	25	59.18
	408	0	486	110	

In the above table, taking the year 1995 as the origin, the time variables (x) are calculated. Now, since $\sum x = 0$ and n, the number of years is 11.

So, $$\sum y = na \quad \text{and} \quad \sum xy = b\sum x^2,$$

which gives $11a = 408.$

$\therefore$ $a = 37.091$

and $110b = 486.$

$\therefore$ $b = 4.418.$

So, equation to the trend line is

$$y = 37.091 + 4.418x. \qquad \text{(i)}$$

Now the trend value for the year 1990 will be obtained by putting $x = -5$ in (i).

$$y_{1990} = 37.091 + 4.418(-5) = \text{ Rs. } 15.$$

In a similar way, the trend values for other years can be found.

The graph of the above data with corresponding trend line is given below :

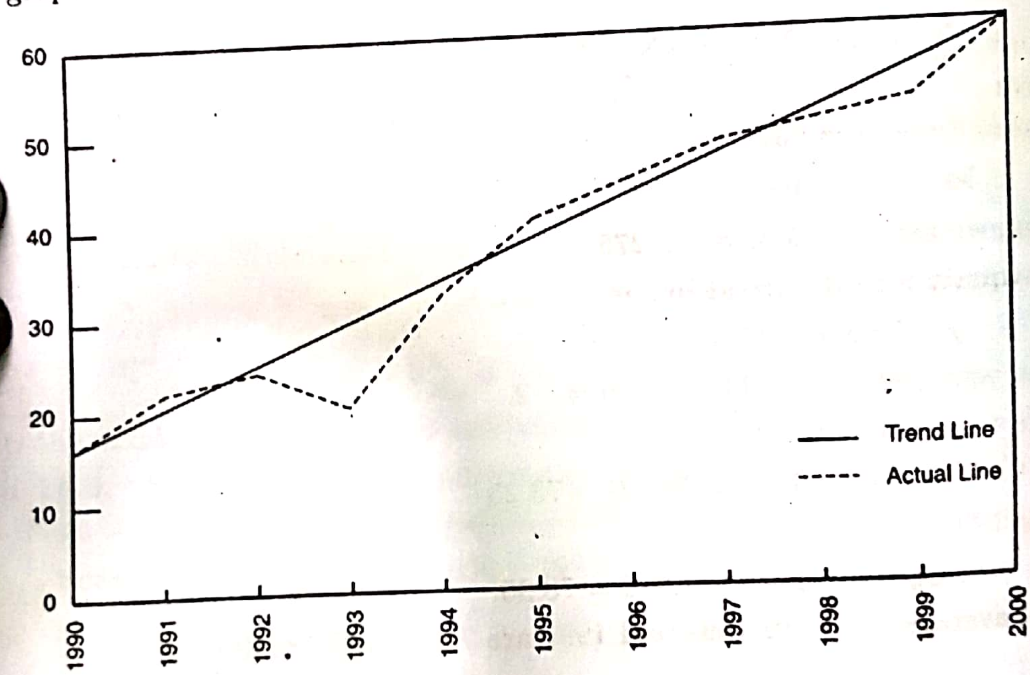

Fig. 15.6

The initial point of the trend line is $(-5, 15)$ and the final point is $(5, 59.18)$, which correspond to $(1990, 15)$ and $(2000, 59.18)$ respectively.

Problem 15.8. Below are given the figures of average number of birds (in thousands) in a poultry for several given years.

Year :	1992	1993	1995	1996	1999	2000	2002
No. of birds ('000) :	68	72	73	80	81	85	78.

Fit a straight line by the method of least squares and determine the trend values in 1994 and 1998. Find also the monthly increase in production.

Solution : Let us take the year 1996 as the origin.

Year	Average stock (y)	x (1996 as the origin)	xy	x^2
1992	68	−4	−272	16
1993	72	−3	−216	9
1995	73	−1	−73	1
1996	80	0	0	0
1999	81	3	243	9
2000	85	4	340	16
2002	78	6	468	36
	537	5	490	87

Here $\sum x = 5$, $\sum xy = 490$, $\sum x^2 = 87$, $n = 7$. Setting the values in the normal equations,

$$\sum y = na + b\sum x \quad \text{(i)}$$

$$\sum xy = a\sum x + b\sum x^2, \quad \text{(ii)}$$

we have

$$7a + 5b = 537 \quad \text{(iii)}$$

$$5a + 87b = 490. \quad \text{(iv)}$$

Solving, we get $a = 75.8$, $b \doteq 1.275$.

$\therefore$ the equation to the trend line is

$$y = 75.8 + 1.275x.$$

For the year 1994, $x = 1994 - 1996 = -2$.

The trend value is

$$y_{1994} = 75.8 + 1.275(-2) = 73.25$$

and for 1998, $x = 2$.

$$\therefore \quad y_{1998} = 75.8 + 2 \times 1.275 = 78.35.$$

So, the average stocks in 1994 and 1998 are 73250 and 78350 respectively.

Note :

(1) If the trend line be $y = a + bx$, where the unit of time (x) is 1 year, then the equation of trend for monthly average is $y = \frac{a}{12} + \frac{b}{12}x$, where unit of x is 1 year. If x be 1 month $= \frac{1}{12}$ year, the equation becomes $y = \frac{a}{12} + \frac{b}{144}x$, unit x is 1 month.

(2) If the time unit (x) be 1 year, then b gives the average change in a year, so the average monthly change in a year is $\frac{b}{12}$.

Merits :

(1) This method is based on mathematical measurement, so it is almost free from subjectivity.

(2) Trend values can be determined for the total period and also at the end points which is not possible through the method of moving average.

(3) This method is dependable for estimation of future trend.

Limitations :

(1) The choice of trend, whether linear or parabolic or exponential, etc., is somewhat subjective.

(2) Estimation is only based on long-term variation like trend but in this method the effect of other variations like cyclical fluctuation, seasonal variation, irregular variations are all neglected.

(3) This method is complicated in comparison with other methods for trend.

(4) When some more data is added, the extension of the method is not possible. The whole method is to be followed afresh from the beginning.

(5) Sometimes a free-hand drawing drawn with care is more effective than the well-guarded mathematically computed 'best fit line', as the former one is the output of the experience of statistician.

(b) Non-Linear Trend of Second Degree

In the parabolic trend, the equation is taken in the form

$$y = a + bx + cx^2, \tag{4}$$

where a is the intercept on y-axis, b is the slope of the parabola at the origin and c is the rate of change of the slope.

Taking x as the time variable and y as the data for the corresponding year, we can find the constants a, b, c using three normal equations,

$$\sum y = na + b\sum x + c\sum x^2 \tag{5}$$

$$\sum xy = a\sum x + b\sum x^2 + c\sum x^3 \tag{6}$$

$$\sum x^2y = a\sum x^2 + b\sum x^3 + c\sum x^4, \tag{7}$$

where n is the number of years or data over which the summation is made.

Problem 15.9. Fit a non-linear trend for the second degree polynomial of the following data, on the highest score in statistics in a college :

Year :	1988	1989	1990	1991	1992	1993	1994
Marks :	88	91	86	89	94	91	94.

Solution : Let us take 1991 as the origin.

Year	Marks (y)	x	x^2	x^3	x^4	xy	x^2y
1988	88	−3	9	−27	81	−264	792
1989	91	−2	4	−8	16	−182	364
1990	86	−1	1	−1	1	−86	86
1991	89	0	0	0	0	0	0
1992	94	1	1	1	1	94	94
1993	91	2	4	8	16	182	364
1994	94	3	9	27	81	282	846
	633	0	28	0	196	26	2546

Here, $n = 7$, $\sum x = 0$, $\sum x^3 = 0$.

So, the normal equations are reduced to the form :

$$\sum y = na + c\sum x^2 \quad \text{(i)}$$

$$\sum xy = b\sum x^2 \quad \text{(ii)}$$

$$\sum x^2 y = a\sum x^2 + c\sum x^4. \quad \text{(iii)}$$

Substituting the values we get,

$$7a + 28c = 633$$

$$28b = 26$$

$$28a + 196c = 2546.$$

Solving the equations, we get

$$a = 89.76,\ b = 0.93,\ c = 0.17.$$

So, the trend line is

$$y = 89.76 + 0.93x + 0.17x^2.$$

Note : It is important to note the suitability of the data where the non-linear second degree trend is applicable. A mere example is sufficient to be convinced.

x	$y = 2 + 3x + 4x^2$	First Difference	Second Difference
1	9	–	–
2	24	15	–
3	47	23	8
4	78	31	8
5	117	39	8
6	164	47	8

The above method is known as the method of second difference. If the second difference of the given data is constant, it will be an ideal case for parabolic trend.

In case of a secular trend, the distribution is not linear. So in that case, this second degree trend will be appropriate.

(c) Exponential Trend

Let the trend equation be of the form $Y_t = ab^t$.

Then $\log Y_t = \log a + t \log b$.

Putting $\log Y_t = Y$, $\log a = A$, $\log b = B$, the equation will be reduced to the form $Y = A + Bt$.

Now proceeding in a similar way like linear trend equation, we determine A and B through normal equations.

Problem 15.10. On a certain trade fair, the number of visitors are recorded as follows :

Day :	1	3	5	7	9
Visitors '000 :	2.5	5.1	11.2	23.4	46.6.

Using exponential trend find the number of visitors attended on 11th day, the closing day. How many visitors came on the day before last?

Solution : Let the exponential trend equation be

$Y_t = ab^t$, origin be day 5 and unit of t is 2,

which is reduced to the form :

$$Y = A + Bt,$$

where $A = \log a$, $B = \log b$ and $Y = \log Y_t$.

Then normal equations are

$$\sum Y = nA + B\sum t \quad (1)$$

$$\sum tY = A\sum t + B\sum t^2. \quad (2)$$

Day	$t = \frac{\text{Day-5}}{2}$	No. of visitors ('000) Y_t	$Y = \log Y_t$	tY	t^2
1	−2	2.5	0.3979	−0.7958	4
3	−1	5.1	0.7076	−0.7076	1
5	0	11.2	1.0492	0	0
7	1	23.4	1.3692	1.3692	1
9	2	46.6	1.6684	3.3368	4
	0		5.1923	3.2026	10

Putting the values in (1) and (2),

$5.1923 = 5A + B \times 0.$

$\therefore \quad A = \frac{5.1923}{5} = 1.038946$ and $3.2026 = A \times 0 + B \times 10$

$\therefore \quad B = 0.32026.$

Hence, $a = \text{antilog}\,(1.03846) = 10.925$

$b = \text{antilog}\,(0.32026) = 2.09.$

So the exponential trend equation is

$Y_t = 10.925 \times 2.09^t.$

On Day 11, $t = 3$. So the estimated visitors will be (Y_t in thousand)

$= 10.925 \times (2.09)^3 = 99.738$ i.e., $99.738 \times 10^3 = 99738$ (nearly).

On 10th day $t = 2.5$, so visitors on the day before last will be (Y_t in thousand)

$= 10.925 \times (2.09)^{2.5} = 68.99$, i.e., $68.99 \times 10^3 = 68990.$

Reduction of Trend Equation from One Unit to Other

Let the yearly trend equation on yearly total be $Y_t = a + bt$, where t is the yearly time unit, with base year 2009. Here a and b are the constants on the basis of yearly total.

If we like to convert the equation to half yearly trend equations, we are to divide the constants a and b by 2. So trend equation will be

$$Y_t = \frac{a}{2} + \frac{b}{2}t,$$

where unit of time be 1 year. So for half yearly time unit, trend equation will be

$$Y_t = \frac{a}{2} + \frac{b}{2} \cdot \frac{t}{2}.$$

Further to note that the origin of time unit is just at the end of sixth month for the above equation.

So if the origin of time be considered as the middle of the first half, the equation will be

$$Y_t = \frac{a}{2} + \frac{b}{2} \cdot \frac{1}{2}\left(t - \frac{1}{2}\right).$$

Similarly, for the case of origin to be considered at the middle of second half it will be

$$Y_t = \frac{a}{2} + \frac{b}{2} \cdot \frac{1}{2}\left(t + \frac{1}{2}\right).$$

In a similar way, the yearly trend equation when converted to quarterly trend equation, it will be converted to

$$Y_t = \frac{a}{4} + \frac{b}{4} \cdot \frac{t}{4}.$$

But in this case, the origin is at the middle of the year.

So, for proper centring of trend values the origin may be considered either at half a quarter to the right or to the left.

Hence, taking origin in the middle of second quarter, the equation will be

$$Y_t = \frac{a}{4} + \frac{b}{4} \cdot \frac{1}{4}\left(t - \frac{1}{2}\right)$$

and the equation when origin is taken at the middle of third quarter

$$Y_t = \frac{a}{4} + \frac{b}{4} \cdot \frac{1}{4}\left(t + \frac{1}{2}\right).$$

So monthly trend equation obtained from yearly trend equation with origin at the middle of June is

$$Y_t = \frac{a}{12} + \frac{b}{12} \cdot \frac{1}{12}\left(t - \frac{1}{2}\right) \quad \text{and} \quad Y_t = \frac{a}{12} + \frac{b}{12} \cdot \frac{1}{12}\left(t + \frac{1}{2}\right)$$

with origin in the middle of July.

Problem 15.11. Fit a straight line trend equation by the method of least squares and estimate the trend value of membership in the cultural organisation in 2012.

Year :	2005	2006	2007	2008	2009	2010	2011
No. of members :	380	400	650	720	870	930	980.

Calculate the quarterly trend equation from the yearly trend equation. Also find the quarterly trend value. Find the equation with origin at 2nd quarter of 2008; Calculate trend values. Verify the trend value of 2007 from the quarterly trend equation.

Solution : Let the yearly trend equation be $Y = a + bt$ having the origin at 2008.

Year	No. of members (Y)	t	t^2	Yt	Trend values (rounded off result)
2005	380	−3	9	−1140	373
2006	400	−2	4	−800	484
2007	650	−1	1	−650	594
2008	720	0	0	0	704
2009	870	1	1	870	815
2010	930	2	4	1860	925
2011	980	3	9	2940	1035
	4930	0	28	3080	4930*

* Fractional value adjusted.

Since $\sum t = 0$, normal equations are

$\sum Y = na,\ n = 7$ and

$\sum Yt = b\sum t^2.$

So, $7a = 4930,$ and $a = 704.286.$

$28b = 3080,$ $b = 110.$

So, the trend equation will be

$Y = 704.286 + 110t,$

where unit of t is 1 year and constants a and b are calculated from yearly total, origin is 2008.

Hence, quarterly trend equation with unit of time being quarter of a year is

$$Y = \frac{704.286}{4} + \frac{110}{4} \cdot \frac{t}{4}$$

or, $Y = 176.07 + 6.875t,$

t is measured quarterly and origin is the end of June 2008.

Quarterly trend value $= \frac{b}{4} = 27.5.$

Equation with origin at the middle of 2nd quarter of 2008, is

$$Y = 176.07 + 6.875\left(t - \frac{1}{2}\right).$$

Trend value at the middle of 2007 is obtained by putting $t = -\frac{7}{2}$.

So Y (at middle of 2007) $= 176.07 + 6.875\left(-\frac{7}{2} - \frac{1}{2}\right) = 148.57$

So yearly trend value in 2007 $= 148.57 \times 4 = 594.28 \simeq 594$ (verified).

Problem 15.12. Yearly trend equation with yearly total is $Y = 312 + 24t$, where base year is 2012, and unit of time is $\frac{1}{2}$ year. Find the monthly trend equation with monthly unit of time, the origin being at May 16.

Solution : Here yearly trend equation is

$$Y = 312 + 24t,$$

where constants are yearly total and time unit is $\frac{1}{2}$ year.

Hence, monthly trend equation with monthly total is

$$Y = \frac{312}{12} + \frac{24}{12}t,$$

where unit of t is $\frac{1}{2}$ year.

So when unit of t is a month, the equation will be

$$Y = 26 + \frac{2}{6}t \quad [t \text{ is divided by } 6]$$

$$\therefore \quad Y = 26 + \frac{1}{3}t.$$

Here the origin is at the end of June. So if the origin is shifted at 16th May, t should be replaced by $\left(t - \frac{3}{2}\right)$.

The required trend equation with monthly total, monthly unit of time and origin at middle of May, is

$$Y = 26 + \frac{1}{3}\left(t - \frac{3}{2}\right).$$

Problem 15.13. Given an annual trend equation $Y = 42 + 24t$, unit of t being 1 year, transform the monthly trend equation with unit of time as 1 month, with origin in the middle of June and July.

Solution : The yearly trend equation with yearly total is $Y = 42 + 24t$, unit of t being 1 year.

If we divide the right side of the trend equation by 12, the output Y will give the monthly output as

$$Y = \frac{42}{12} + \frac{24}{12}t \quad \text{or,} \quad Y = 3.5 + 2t.$$

Here the unit of t is 1 year.

So monthly trend equation with unit of time as 1 month is

$$Y = 3.5 + \frac{2}{12}t.$$

The origin of the above equation is middle of the year, i.e., the end of June or the beginning of July.

Hence, if the origin be shifted to half of the month of June or half of the month of July, we get the following equations :

$$Y = 3.5 + \frac{1}{6}\left(t - \frac{1}{2}\right), \text{ origin } \frac{1}{2} \text{ of June}$$

$$Y = 3.5 + \frac{1}{6}\left(t + \frac{1}{2}\right), \text{ origin } \frac{1}{2} \text{ of July.}$$

roblem 15.14. Given that the following trend equation $Y_t = 360 + 4.2t$, the origin being 2006, unit of time being 1 year and Y_t being the unit of production, shift the origin to 2008–09. What change to be noted when $Y_t = 120 + 40t + 30t^2$?

Solution : 2008–09 means we will take 1st January, 2009 and origin at 2006 means it would be 1st July 2006. So the time unit to be shifted 2.5 years.

ce, Y_t will be given by

$$Y_t = 360 + 4.2(t + 2.5) = 370.5 + 4.2t,$$

unit of time being 1 year.

case when $Y_t = 120 + 40t + 30t^2$, the necessary correction will be

$$Y_t = 120 + 40(t + 2.5) + 30(t + 2.5)^2 = 407.5 + 190t + 30t^2.$$

te : For a shift of λ unit of time $Y_t(t)$ is to be shifted to $Y_t(t+\lambda)$, i.e., if $Y_t = ab^t$ and shift is λ unit, then final trend equation will be $Y_t = ab^{(t+\lambda)}$.

Measurement of Seasonal Variation

We have already discussed about this variation. Some more examples may elucidate the idea. In July to September generally we see that sale of textbooks attains its peak in comparison to that in the rest period of the year. The effect of seasonal variation shows a considerable variation from its normal flow throughout the year. To a bank manager, businessman or an industrialist it is important to know seasonal component so as to plan their future steps.

Seasonal index : It is an indicator expressed in percentage measured quantitatively the extent of variation of the seasonal component in the time series.

The absolute value of the seasonal variation can be measured through additive model, but better measurement of such variation is the measurement of seasonal index. While measuring the index, if the data is given quarterly, we are to consider the season as a quarter of a year. In that case, 4 such calculations for each quarter are to be made. Similarly, for a monthly data, 12 indices for each month are to be calculated.

The criteria for good measurement of seasonal variation are the following :

(i) To isolate the seasonal variation only and to remove the influence of other components like trend, cyclical fluctuation or irregular fluctuation.

(ii) To recognize slowly changing seasonal pattern which may be included in the series and the index should be modified to cope up with the change.

Generally, there are four common methods for the measurement of seasonal index :

(1) Method of simple averages (weekly, monthly, quarterly)

(2) Ratio-to-trend method

(3) Ratio-to-moving average method

(4) Link relative method.

1. Method of simple averages

(i) First of all the data should be arranged row-, and column-wise.

(ii) The total of each quarter or each month is to be taken.

(iii) The arithmetic mean is now calculated by dividing each quarterly/monthly total by the number of years.

(iv) Next the average of all the A.M. is calculated, which may be termed as **Grand Average.**

(v) Now express each quarterly/monthly average as a percentage of Grand Average. This number is termed as seasonal index.

(vi) Average seasonal variation can also be calculated by subtracting the grand average from each individual average.

Problem 15.15. Calculate the average seasonal variation and the seasonal index from the following table :

Total production of cement ('00,000 tons)

	Quarter			
Year	I	II	III	IV
1990	34	32	31	36
1991	37	34	33	41
1992	43	40	33	43

Solution : **Calculation for average seasonal variation**

Quarter / Year	Production ('00,000 tons)				
	I	II	III	IV	Total
1990	34	32	31	36	
1991	37	34	33	41	
1992	43	40	33	43	
Total	114	106	97	120	480
Mean	38	35.33	32.33	40	145.66
Grand Average = 145.66 ÷ 4 = 36.415					
Average Seasonal Variation	1.585	−1.085	−4.085	3.585	0

Procedure : *Average seasonal variation (ASV) of each quarter is calculated by subtracting the grand average from respective mean of each quarter. As for example,* 38 − 36.415 = 1.585, 35.33 − 36.415 = −1.085 *and so on. Ultimately we will get the total seasonal variation as* 0. *If in case, where this sum is not coming zero, slight adjustment in ASV should be made in order to make the sum zero.*

Seasonal Index = (Mean/Grand Average) × 100

∴ SI of Quarter I = (38/36.415) × 100 = 104.35

" " " II = (35.33/36.415) × 100 = 97.02

" " " III = (32.33/36.415) × 100 = 88.78

" " " IV = (40/36.415) × 100 = 109.85.

Note : (1) Here also some necessary adjustment in the decimal places should be made so that the sum of the seasonal indices becomes 400.

(2) We observe that the 4th quarter production value has highest and 3rd quarter production value has lowest seasonal effect.

Merits :

(1) The method of simple average is really simple with less mathematical effort.

(2) This is a suitable method for the measurement of seasonal variation where there is no trend in the distribution.

Limitations :

(1) In practical point of view, most of the economic series have a trend, so in that case seasonal index calculated through this method is an index of seasonal variation with trend also.

(2) The effect of cyclical fluctuation may not be eradicated through averaging process. So, the utility of this method is not much.

2. Ratio-to-trend method

This is rather an improved method than that of simple averages for finding the index number for seasonal variation. From a time-series data, first the trend is calculated by method of least squares. Then the individual observed values are divided by the corresponding trend values.

As from the multiplicative model, $Y = T \times S \times C \times I$, so $Y/T = S \times C \times I$.

In this way the effect of trend values are eliminated. Then the trend-free values are multiplied by 100. Again, the cyclical variation can be removed by taking the average of monthly data for all years taken together.

To determine the seasonal index through this method, the following steps are to be followed :

Step 1

To follow the method of least squares for trend values.

Step 2

Next the original month-wise data are divided by the corresponding trend values and multiplied by 100.

Step 3

As in the step 2, the effect of trend is removed, steps are to be taken to remove the effect of cyclical and irregular fluctuations. Initially, if in the data there are abnormally high or low values, we may neglect these values and calculate the A.M. of the rest, which is termed as modified mean. In such cases we may also find median as in case of median those extreme values are automatically left out. Now we are to calculate mean/median/modified A.M. of the above percentage figures for each quarter/month. Through this process cyclical and irregular variations are removed. This average is the seasonal index for that month.

Step 4

The sum of the indices of 12 months is 1200. So if we observe that the calculated sum is not 1200, then to adjust the constant factor as 1200/(Total of indices for 12 months). If the indices for quarter are found, then

$$\text{Adjustment factor} = 400/(\text{Total of indices for 4 quarters}).$$

Hence follows the final seasonal indices.

Synoptically, it may be thoug it that the trend values for each month is determined first, then the original values of e ich month is divided by trend values and then express that ratio to trend through per entage. Now the values for month are arranged and finally those averages are so adjusted that their sum becomes 1200 and in case of quarterly average the sum is 400.

Problem 15.16. Find the seasonal variations by ratio-to-trend method from the following data :

	Quarter			
Year	**I**	**II**	**III**	**IV**
1996	32	46	38	36
1997	36	62	48	42
1998	42	70	64	62
1999	58	82	68	64
2000	66	84	72	68

Solution : To calculate trend values, we will find at first yearly data and then convert it to quarterly mode.

Calculation for yearly trend

Year	Yearly total	Quarterly average (y)	Deviation from the mid-year (x)	x^2	xy	Trend value
1996	152	38	−2	4	−76	39
1997	188	47	−1	1	−47	48
1998	178	59.5	0	0	0	57
1999	272	68	1	1	68	66
2000	290	72.5	2	4	145	75
		285	0	10	90	285

Let the equation to the trend line be $y = a + bx$,

where $a = \frac{\sum y}{n} = \frac{285}{5} = 57$

$$b = \frac{\sum xy}{\sum x^2} = \frac{90}{10} = 9.$$

So, the trend line is

$$y = 57 + 9x.$$

Here the annual increment is 9, so quarterly increment $= \frac{9}{4} = 2.25$.

Now to calculate the quarterly trend, we are to take the trend value of each year at the middle of 2nd and 3rd quarters.

Therefore, trend at 2nd quarter

$$= \text{yearly trend} - \frac{1}{2}\text{ (quarterly increment)}$$

and trend at 3rd quarter

$$= \text{yearly trend} + \frac{1}{2}\text{ (quarterly increment).}$$

Similarly, at 1st and 4th quarters the trend values are yearly trend $\mp\frac{3}{2}$ (quarterly increment) respectively.

In 1996, trend value = 39. Hence, the trend values for 1st, 2nd, 3rd and 4th quarters are 39−1.5(2.25), 39−0.5(2.25), 39+0.5(2.25) and 39+1.5(2.25) respectively.

In a similar way, trend values at all quarters in each year are to be tabulated.

Trend values

Year \ Quarter	I	II	III	IV
1996	35.625	37.875	40.125	42.375
1997	44.625	46.875	49.125	51.375
1998	53.625	55.875	58.125	60.375
1999	62.625	64.875	67.125	69.375
2000	68.625	73.875	76.125	78.375

Now the given values of the original table are to be expressed as the percentage of the corresponding trend values.

In 1996 (I Qr) it will be $\frac{32 \times 100}{35.625} = 89.82$; in 1996 (II Qr) it is $\frac{46 \times 100}{37.875} = 121.45$.

In a similar way we will compute the following table :

Percentage of trend values

Year	I Qr	II Qr	III Qr	IV Qr
1996	89.82	121.45	94.70	84.95
1997	80.67	132.27	97.71	81.75
1998	78.32	125.28	110.11	102.69
1999	92.61	126.40	101.30	92.25
2000	96.17	113.70	94.58	86.76
Total	437.59	619.10	498.40	448.40
Average	87.52	123.82	99.68	89.68 (400.7)
Adjusted Seasonal Index	87.37	123.60	99.51	89.52

Note : Here the average of I Qr = 87.52.

Total of averages = 400.7.

Hence, the adjustment factor = $\frac{400}{400.7}$.

So, the adjusted seasonal index of I Qr = $\frac{87.52 \times 400}{400.7} = 87.37$, so on.

Merits :

(1) It is easy to calculate and also it can easily be understood.

(2) In comparison to the method of simple monthly average it is more logical.

(3) In comparison to the method of moving average, this method has additional advantage that the trend value for each month is available here, as there is no case of omission of data.

Limitation :

(1) The ratio-to-trend method cannot follow the actual data so closely as it is done by twelve-month moving average in case where there is a clear picture of cyclical fluctuation in the series. Consequently, we get more biased result for seasonal index in computing through ratio-to-trend method than it is computed through ratio-to-moving average method.

Problem 15.17. A statistician from the data on yearly sales of a cloth merchant got the trend line as $y = 58 + 0.3x$ with origin at the 1st quarter of 1998.

If the quarterly sales (Rs. '000) be y, the time unit (one quarter) be x and seasonal indices for the quarters are 85, 90, 105, 120 respectively, then using multiplicative model, find the estimated sales in each quarter of 2000.

Solution : As the trend equation be $y = 58+0.3x$, then trend values for the quarters of 2000, will be obtained by putting $x = 8, 9, 10, 11$, as there are 8 quarters within the span of two years.

Trend for I Qr in 2000 $= 58 + 0.3(8) =$ Rs. 60.4×1000

for II Qr " " $= 58 + 0.3(9) =$ Rs. 60.7×1000

for III Qr " " $= 58 + 0.3(10) =$ Rs. 61×1000

for IV Qr " " $= 58 + 0.3(11) =$ Rs. 61.3×1000

Now using multiplicative model, we can find the quarterly sales as $T \times S$ in the following tabular form :

Quarter (2000)	Trend	Seasonal index	Quarterly sales (in Rs. '000)
I	60.4	.85	51.34
II	60.7	.90	54.63
III	61	1.05	64.05
IV	61.3	1.20	73.56

So estimated sales in each quarter are Rs. 51,340, Rs. 54,630, Rs. 64,050, Rs. 73,560, respectively.

3. Ratio-to-moving average method

This method is based on the fact that seasonal fluctuation is being removed through moving average over 12 months in case of month-wise data or 4-quarter moving average in case of quarterly data. In this moving averages the trend and cyclical effects are left. To eliminate those effects, the original data is divided by respective moving averages and finally expressed as the percentages of respective moving averages.

Now on averaging those percentages and multiplying by correction factor we get the seasonal index.

For calculation of seasonal index the following steps are to be followed:

Step 1 :

Either 12-monthly or 4-quarterly moving average is calculated according as 12-monthly or 4-quarterly data.

Step 2 :

Express the original 12-monthly/4-quarterly data as a percentage of centred 12-month/4-quarter moving average.

Step 3 :

Arrange chronologically those percentages in a table and calculate the arithmetic mean of those percentage values. Hence, the seasonal indices are obtained,

Step 4 :

Now multiplying each index by corresponding correction factor we get the correct seasonal index for each month/quarter.

The steps so far discussed are all for multiplicative model. When the additive model is used, the steps are as following :

Step 1 :

The moving average (with 12-month or 4-quarter) should be calculated.

Step 2 :

The moving average values are to be subtracted from the original values.

Step 3 :

The deviations are arranged chronologically and the averaging is done. Now the adjustment is to be done to get theseasonal index.

In this additive model, adjustment is taken as the average of the averages with negative sign.

Problem 15.18. Calculate the seasonal index by the method of ratio-to-moving average of the following data :

	I Qr	II Qr	III Qr	IV Qr
1987	27	32	40	26
1988	32	40	26	30
1989	30	35	22	28.

Solution :

Calculation of seasonal indices by ratio-to-moving average method

Year	Quarter	Given data	4-figure moving total	2-item moving total	*4-figure moving average (centred)	**Given data as % of moving average
1987	I	27	–	–	–	–
	II	32	–	–	–	–
			125			
	III	40		255	31.87	125.49
			130			
	IV	26		268	33.50	77.61
			138			

Contd.

Year	Quarter	Given data	4-figure moving total	2-item moving total	*4-figure moving average (centred)	**Given data as % of moving average
1988	I	32		262	32.75	97.71
			124			
	II	40		252	31.50	126.98
			128			
	III	26		254	31.75	81.89
			126			
	IV	30		247	30.87	97.16
			121			
1989	I	30		238	29.75	100.84
			117			
	II	35		232	29.00	120.69
			115			
	III	22	–	–	–	–
	IV	28	–	–	–	–

* 4-figure moving average is obtained by dividing the corresponding 2-item moving total by 8; $255 \div 8 = 31.87$, $268 \div 8 = 33.50$, so on.

** $\frac{40}{31.87} \times 100 = 125.49$, $\frac{26}{33.5} \times 100 = 77.61$, etc.

Calculation of seasonal index

	Percentage to moving average				
Year	I Qr	II Qr	III Qr	IV Qr	Total
1987	–	–	125.49	77.61	
1988	97.71	126.98	81.89	97.16	
1989	100.84	120.69	–	–	
Total	198.55	247.67	207.38	174.77	
Average	99.27	123.83	103.69	87.38	414.17
Seasonal index*	95.88	119.59	100.14	84.39	

* Seasonal index = (Average ÷ Total of average) × 400

$$= \frac{99.27}{414.17} \times 400 = 95.88$$

and so on.

Now we use additive model to solve the above problem.

Year	Quarter	Data	4-figure moving total	2-item moving total	4-quarter moving average	Deviation from original value
1987	I	27	–		–	
	II	32	–		–	
			125			
	III	40		255	31.87	8.13
			130			
	IV	26		268	33.50	−7.50
			138			
1988	I	32		262	32.75	−0.75
			124			
	II	40		252	31.50	8.50
			128			
	III	26		254	31.75	−5.75
			126			
	IV	30		247	30.87	−0.87
			121			
1989	I	30		238	29.75	0.25
			117			
	II	35		232	29.00	6.00
			115			
	III	22	–			
	IV	28	–			

Calculation of seasonal index

	Deviation from original data				
Year	I Qr	II Qr	III Qr	IV Qr	Total
1987	–	–	8.13	−7.50	
1988	−0.75	8.50	−5.75	−0.87	
1989	0.25	6.00	–	–	
Total	−0.5	14.5	2.38	−8.37	
Average	−0.25	7.25	1.19	−4.18	4.01
* Adjustment	−1.00	−1.00	−1.00	−1.01	−4.01
Seasonal index	−1.25	6.25	0.19	−5.19	0

* Adjustment = Average of the averages with negative sign

$$= -(4.01/4) = -1.00.$$

We have already discussed that sometimes median is used instead of arithmetic mean. In case of 12-month moving average, out of total time span initial six months and final six months are left from the final table, median will be the best central value.

Problem 15.19. Calculate seasonal indices by the method of ratio-to-moving average of the following data.

	Month											
	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Sales ('000) (1992)	12	14	15	17	18	16	19	20	18	17	15	14
Sales ('000) (1993)	15	17	18	16	19	17	15	14	18	15	14	12
Sales ('000) (1994)	20	18	16	14	12	15	17	18	16	22	16	24
Sales ('000) (1995)	22	16	18	12	15	17	18	16	12	20	18	14

Solution :

Calculation of 12-month moving average

Year	Month	Sales ('000)	12-month moving total	12-month moving average (non-centred)	2-item moving total	12-month moving average (centred)	Original data as % of moving average
1992							
	Jan	12	–	–	–	–	–
	Feb	14	–	–	–	–	–
	Mar	15	–	–	–	–	–
	Apr	17	–	–	–	–	–
	May	18	–	–	–	–	–
	Jun	16	–	–	–	–	–
			195	16.25			
	Jul	19			32.75	16.375	116.03
			198	16.50			
	Aug	20			33.25	16.625	120.30
			201	16.75			
	Sep	18			33.75	16.875	106.67
			204	17.00			
	Oct	17			33.92	16.960	100.23
			203	16.92			
	Nov	15			33.92	16.960	88.44
			204	17.00			
	Dec	14			34.08	17.040	82.16
1993			205	17.08			
	Jan	15			3.83	16.915	88.68
			201	16.75			
	Feb	17			33.00	16.500	103.03
			195	16.25			
	Mar	18			32.50	16.250	110.77
			195	16.25			

Contd.

Year	Month	Sales ('000)	12-month moving total	12-month moving average (non-centred)	2-item moving total	12-month moving average (centred)	Original data as % of moving average
1993							
	Apr	16			32.33	16.165	98.98
			193	16.08			
	May	19			32.08	16.040	118.45
			192	16.00			
	Jun	17			31.83	15.915	106.82
			190	15.83			
	Jul	15			32.08	16.040	93.52
			195	16.25			
	Aug	14			32.58	16.290	85.94
			196	16.33			
	Sep	18			32.50	16.250	110.77
			194	16.17			
	Oct	15			32.17	16.085	93.25
			192	16.00			
	Nov	14			31.42	15.710	89.11
			185	15.42			
	Dec	12			30.67	15.335	78.25
			183	15.25			
1994							
	Jan	20			30.67	15.335	130.42
			185	15.42			
	Feb	18			31.17	15.585	115.49
			189	15.75			
	Mar	16			31.33	15.665	102.14
			187	15.58			
	Apr	14			31.75	15.875	88.19
			194	16.17			
	May	12			32.50	16.250	73.85
			196	16.33			
	Jun	15			33.66	16.83	89.13
			208	17.33			
	Jul	17			34.83	17.415	97.62
			210	17.50			
	Aug	18			34.83	17.415	103.36
			208	17.33			
	Sep	16			34.83	17.415	91.87
			210	17.50			
	Oct	22			34.83	17.415	126.33
			208	17.33			
	Nov	16			34.91	17.455	91.66
			211	17.58			
	Dec	24			35.33	17.665	135.86

Contd.

Year	Month	Sales ('000)	12-month moving total	12-month moving average (non-centred)	2-item moving total	12-month moving average (centred)	Original data as % of moving average
1995			213	17.75			
	Jan	22			35.58	17.790	123.66
			214	17.83			
	Feb	16			35.50	17.750	90.14
			212	17.67			
	Mar	18			35.00	17.500	102.86
			208	17.33			
	Apr	12			34.49	17.245	69.58
			206	17.16			
	May	15			34.49	17.245	86.98
			208	17.33			
	Jun	17			33.83	16.915	100.50
			198	16.50			
	Jul	18	–	–	–	–	–
	Aug	16	–	–	–	–	–
	Sep	12	–	–	–	–	–
	Oct	20	–	–	–	–	–
	Nov	18	–	–	–	–	–
	Dec	14	–	–	–	–	–

Computation of seasonal indices

Month	1992	1993	1994	1995	Median	Seasonal Index
Jan	–	88.68	13042	123.66	123.66	125.29
Feb	–	103.03	115.49	90.14	103.03	104.39
Mar	–	110.77	102.14	102.86	102.86	104.22
Apr	–	98.98	88.19	69.58	88.19	89.35
May	–	118.45	73.85	86.98	86.98	88.13
Jun	–	106.82	89.13	100.50	100.50	101.83
Jul	116.03	93.52	97.62	–	97.62	98.92
Aug	120.30	85.94	103.36	–	103.36	104.72
Sep	106.67	110.77	91.87	–	106.67	108.08
Oct	100.23	93.25	126.33	–	100.23	101.55
Nov	88.44	89.11	91.66	–	89.11	90.28
Dec	82.16	78.25	135.86	–	82.16	83.24
					1184.37	

Seasonal indices are obtained by multiplying the median by the adjustment factor.

Here sum of the medians = 1184.37.

So adjustment factor $= \dfrac{1200}{1184.37}$.

Hence, S.I. in January $= \dfrac{123.66 \times 1200}{1184.37} = 125.29$, etc.

Merits :

(1) This method is based on mathematical backgrounds and reliable.

(2) This method is considered satisfactory and widely used since the index calculated through this method has less fluctuation than it is computed through ratio-to-trend method.

(3) This method removes the influence of trend and cyclical fluctuation from the seasonal index.

Limitation :

(1) If the data has regular seasonal variation and also uniform periodicity, then only the method works well, otherwise the result is not so much reliable.

4. Link relative method

This method is somewhat complicated. The notion 'link relative' is arising as each data is compared with the previous data. So a link between the two is established, that is why it is known as link relative method. *Here each data is expressed as the percentage of the previous data.*

For calculation of seasonal indices through this method, the following steps are to be followed carefully:

Step 1

Calculate the link relatives (L.R.) of the given seasonal data. Only the first data is left, as there is no data prior to the first.

Step 2

Arithmetic mean (or median) of link relatives for each season (month/quarter) is now calculated.

Step 3

Next to convert those arithmetic means of the link relatives to chain relatives (C.R.) on the basis of the first season (month/quarter).

Step 4

Next chain relative of any time span is obtained by multiplying the L.R. of that span by the C.R. of previous span and dividing by 100.

Step 5

Again, the C.R. of the first season is calculated on the basis of the last season. Thus, we will get two values of C.R. of the first season, one obtained on the basis of the first season and the other obtained on the basis of the last season. There will be a specific difference between the two.

Step 6

The above difference is divided by the number of seasons and the result is multiplied by 1, 2, 3, and so on and is deducted from 2nd, 3rd, 4th seasons and so on respectively, to get the correct chain relatives.

Step 7

Ultimately, we get the seasonal indices for each season by expressing each corrected C.R. as the percentage of the average of them.

Problem 15.20. Calculate seasonal indices of the following data through the method of link relatives:

Quarter	1996	1997	1998	1999
I	10	11.5	11	12
II	8	9.6	10.8	10.3
III	7.5	8	9.5	8.5
IV	9.5	12	10.4	10

Solution :

Calculation through link relative method

Year	Quarter			
	I	II	III	IV
1996	–	80	93.75	126.67
1997	121.05	83.48	83.33	150.00
1998	91.67	98.18	87.96	109.47
1999	115.38	85.83	82.52	117.65
Arithmetic average	$\frac{328.1}{3}$ = 109.37	$\frac{347.49}{4}$ = 86.87	$\frac{347.56}{4}$ = 86.89	$\frac{503.79}{4}$ = 125.95
Chain relative	100	$\frac{100 \times 86.87}{100}$ = 86.87	$\frac{86.87 \times 86.89}{100}$ = 75.48	$\frac{75.48 \times 125.95}{100}$ = 95.07
Corrected chain relative	100	86.87 – 0.995 = 85.875	75.48 – 2 × 0.995 = 73.49	95.07 – 3 × 0.995 = 92.085
Seasonal index	$\frac{100 \times 100}{87.86}$ = 113.82	$\frac{85.875 \times 100}{87.86}$ = 97.74	$\frac{73.49 \times 100}{87.86}$ = 83.64	$\frac{92.085 \times 100}{87.86}$ = 104.81

For 1996 (Qr II), link relative $= \frac{8}{10} \times 100 = 80$,

For 1996 (Qr III), link relative $= \frac{7.5}{8} \times 100 = 93.75$,

For 1997 (Qr I), link relative $= \frac{11.5}{9.5} \times 100 = 121.05$, and so on.

Chain relative (C.R) for the first quarter = 100
(on the basis of first quarter)

Chain relative for the second quarter

$$= (\text{Average of second quarter} \times \text{C.R. of first quarter}) \times \frac{1}{100}$$

$$= \frac{100 \times 86.87}{100} = 86.87$$

Chain relative for the third quarter

$$= (\text{Average of third quarter} \times \text{C.R. of second quarter}) \times \frac{1}{100}$$

$$= \frac{86.89 \times 86.87}{100} = 75.48 \text{ and so on.}$$

New chain relative for the first quarter

$$= (\text{chain relative of last quarter} \times \text{average of first quarter}) \times \frac{1}{100}$$

$$= \frac{95.07 \times 109.37}{100} = 103.98.$$

$\therefore$ Adjustment factor

$$= \frac{\text{Difference of chain reltive of first quarter}}{4} = \frac{103.98 - 100}{4} = 0.995.$$

Adjusted chain relative for first quarter = 100,

for second quarter, C.R. = 86.87 − 0.995 = 85.875

for third quarter, C.R. = 75.48 − 2 × 0.995 = 73.49

for fourth quarter, C.R. = 95.07 − 3 × 0.995 = 92.085.

Then average of corrected chain relative

$$= \frac{1}{4}(100 + 85.875 + 73.49 + 92.085) = 87.86$$

$$\text{Seasonal index} = \frac{\text{Corrected chain relative} \times 100}{\text{Average of corrected chain relative}}$$

Merit :

(1) It is notable that where the seasonal pattern is more or less fixed or a clear picture of seasonal pattern occurs link relative method can be used.

Limitations :

(1) Too much mathematical calculations are involved in the computation.

(2) The logical argument regarding the steps to be followed, cannot be easily understood.

(3) The method is unable to eliminate other effect like trend, cyclical fluctuations, etc.

Note 1. The method of monthly average is applicable where there is no trend. If the seasonal variation has the dominating role within the data and consequently the trend and cyclical fluctuations are negligible, the method is effective.

Note 2. Accounting for the merits and demerits of ratio-to-trend method and ratio-to-moving average method, the later method is better. So in general, both from theoretical and practical point of view, **ratio-to-moving average is prescribed.**

Note 3. In computation of seasonal variation, we take the average of seasonal averages, to eliminate random and cyclical effects. Here we have used both the arithmetic mean and the median as the representative data. But we should take care of using any of the two. When the number of data is highly large, arithmetic mean is used and when the period is short, the use of median is recommended.