# Chapter 1 Acceptance Sampling Plans

### 1.1 Introduction

Statistical Quality Control aims at drawing reliable inferences about the quality of a manufactured product by making use of appropriate statistical tools. Statistical Process Control and Acceptance Sampling are two branches of statistical quality control differentiated based on the time of inspection. Statistical process control applies statistical techniques to determine whether the manufacturing process is functioning as per the given quality standards. It is mainly used in manufacturing industries for continuous improvement in quality standards. Statistical process control monitors and improves the production process by checking the quality standards and helps to achieve process stability. It is pertinent to note that some variation in quality standards is inevitable in any production process. This variation is classified into two categories. The first kind of variation, called variation due to allowable causes is intrinsic random variation created by the inherent characteristic of the process which cannot be fully eliminated. If this variation is present in the process, the process is said to be "in control". The second kind of variation called variation due to assignable causes is due to some identifiable causes such as glitch (malfunction), improper arrangements, deterioration (wear and tear), etc. If one or more assignable cause of variation is present then the process is said to be "out of control".

The process control aims at identification of any variation due to assignable causes at the earliest in order to take appropriate action to rectify the problem thereby permitting the production to continue without disruptions when the process is in control.

Control charts (or Shewhart charts) are one of the basic and important statistical tools used to achieve and maintain the process stability. Control charts help practitioners to monitor the process periodically, examine the sample output, and take preventive and corrective action whenever necessary. The basic components of control charts are as follows:

- a center line which is the mathematical average of a statistic based on all the samples
- lower and upper control limits computed on taking into account the common cause of variations
- performance data plotted over a given period of time.

There are two types of control charts, namely

- variable control charts
- attribute control charts.

Variable control charts are used when the quality characteristic under study can be expressed in the continuous scale of measurement like width, height, length, etc.  $\overline{X}$  and R charts are most widely used variable control charts.  $\overline{X}$  and S charts, CUSUM (cumulative sum chart), EWMA (exponentially weighted moving average chart) are some control charts meant for variables.

Attribute control charts are used when the quality characteristic under study cannot be measured in any quantitative scale. They are applicable for situations where characteristic can be categorized into conforming or non-conforming, defective or not defective, success or failure etc. Some popular attribute control charts widely in use are p chart, np chart, u chart and c chart.

Acceptance sampling plans help us to examine whether the manufactured products meet the pre-specified quality levels. They are predominantly used in statistical quality control when complete inspection of the manufactured products is not possible for various reasons like the manufactured products being destructive in nature, inspection may be a time-consuming process. Ultimately, acceptance sampling plans help us to assess the quality level of the product based on sampled items. Quality standards can be studied in two different ways. If the manufactured product is classified into one of the well-defined categories (like 'defective' and 'non defective') then the sampling plan is called sampling plan for attributes. On the other hand, if quality requirements are assessed through measurements (like radius, length, etc.,) then the sampling plan is referred to as sampling plan for variables. This thesis focuses on sampling plans for variables.

According to Dodge (1969), the theory of acceptance sampling plan is categorized into four types and they are listed below:

- lot by lot sampling with attributes inspection
- lot by lot sampling with measurements on variables
- continuous sampling flows of units with inspection of attributes
- special purpose plans like, chain sampling, skip lot sampling, reliability sampling, etc.

To apply lot-by-lot sampling for both attributes and variables, the following conditions are to be satisfied:

- it is essential that the manufacturing process is stable
- a lot should be homogeneous
- there is no process variation
- the probability of producing a defective unit is constant
- random samples are selected from homogenous lot.

Continuous sampling plans are the inspection procedures developed for situations involving continuous production processes where the formation of lots for lot-by-lot acceptance may be unviable. Special purpose sampling plans are inspection procedures defined under the following conditions:

- the production process is steady so that the results of past, current and future lots are largely indicative of a continuing process
- the lots are submitted considerably in the order of their production
- inspection is by attributes with quality level defined in terms of a fraction defectives.

#### Sampling Plan, Sampling Scheme and Sampling System

A sampling plan describes the manner in which measurements are to be taken on the sampled units along with the criteria for acceptance or rejection of the lot. ANSI/ASQC standard A2 (1987) mentions that an acceptance sampling plan states the sampling rules to be used with the acceptance or rejection criteria.

Sampling scheme is a collection of sampling plans. ANSI/ASQC standard A2 (1987) defines sampling scheme as a specific set of rules which usually

consists of sampling plan in which sample size, lot size and acceptance criteria are related.

The difference between a sampling plan and a sampling scheme was described by Hill (1962). According to Hill (1962), a sampling scheme describes an overall strategy which specifies the way of using sampling plans from a collection of sampling plans and operations included in a standard. ISO-3534-2 (2006) defines acceptance sampling system as a collection of sampling plans or sampling scheme together with criteria for choosing appropriate plans or schemes.

#### **Probability Distributions in Acceptance Sampling**

The characteristics of an acceptance sampling plan are studied mainly with the help of probability distributions which involve certain parametric values. For example, in sampling plans for attributes, distributions like binomial, Poisson, hyper-geometric, etc. play important roles. In the case of acceptance sampling plans for variables, distributions like normal, exponential, gamma, weibull, lognormal, etc., find wide applications.

Schilling and Neubauer (2008) stated the following conditions for attribute sampling plans under which binomial, Poisson, and hypergeometric distributions can be used.

- Binomial distribution is more suitable in situations where sampling is done from a finite lot or from an infinite population either with or without replacement whenever  $\frac{n}{N} \le 0.1$  where *n* and *N* are sample and lot sizes respectively.
- Poisson distribution is suitable for situations which focus on number of defects instead of the number of defective units. Poisson distribution can be applied whenever np < 5 where p is a fraction of defective units and n is the sample size.
- Hypergeometric distribution is more suitable for evaluating the probability of acceptance when sampling is made from a finite lot without replacement.

In variable sampling plans, several continuous distributions like normal, exponential and Weibull play an important role even though they are complicated and restrictive in their use. For the sampling plan based on attributes, it is not necessary to know the shape or parameters of the distribution of any measurements but this is not true for variable sampling plans. Under variable sampling plan, the distribution of measurements is usually continuous and needs a specification of shape and parameters such as measures of location and spread.

Normal distribution plays a vital role and its impact is there in every area of statistics. This is true for acceptance sampling where it forms the basis of a large number of variable acceptance sampling plans and has infused other areas of sampling plans.

The normal distribution is completely specified by two parameters  $\mu$  (mean) and  $\sigma$  (standard deviation). There is no closed form formula for the distribution function of normal distribution but values of the distribution function can be obtained from tables of the standard normal distribution.

The exponential distribution is used widely in evaluating acceptance plans for life testing and reliability since this distribution has a renowned constant failure rate. That is, the probability of future failure is constant regardless of how long a unit has been in operation. In life test distribution for units, the constant failure rate is denoted by  $\lambda = \frac{1}{\mu}$  where  $\mu$  is the mean value. In exponential distribution, mean and standard deviation are equal.

Weibull distribution is one of the distributions used in designing acceptance sampling plans for life testing. It is characterized by scale and shape parameters denoted by  $\theta$  and  $\beta$  respectively. The density function of Weibull distribution is given by

$$f(x) = \frac{\beta}{\theta} \left(\frac{x}{\theta}\right)^{\beta-1} \exp\left[-\left(\frac{x}{\theta}\right)^{\beta}\right], \ x \ge 0$$

where  $\theta > 0$  is the scale parameter and  $\beta > 0$  is the shape parameter. The mean and variance of the Weibull distribution are

$$\mu = \theta \, \Gamma \! \left( 1 + \frac{1}{\beta} \right)$$

and

$$\sigma^{2} = \theta^{2} \left[ \Gamma \left( 1 + \frac{2}{\beta} \right) - \Gamma \left( 1 + \frac{1}{\beta} \right)^{2} \right],$$

respectively.

In addition to these distributions, gamma distribution, half logistic distribution, Birnbaum-Saunders distribution, Rayleigh distribution, exponentiated Freche't distribution, etc. have been widely used in designing various acceptance sampling plans for variables.

# **1.2** Types of Acceptance Sampling Plans

In this section, operational details of

- a. Single Sampling Plan
- b. Double Sampling Plan
- c. Multiple Sampling Plan
- d. Sequential Sampling Plan
- e. Skip lot Sampling Plan
- f. Continuous Sampling Plan
- g. Chain Sampling
- h. Tightened Normal Tightened Sampling Plan
- i. Time Truncated Acceptance Sampling Plan
- j. Reliability Sampling Plan

are explained.

#### a. Single Sampling Plan

The single sampling plan is one of the most popular and simplest sampling plans available in the literature. It is classified into two types namely,

- single sampling plans for attributes
- single sampling plans for variables.

#### **Single Sampling Plans for Attributes**

The operational procedure of the sampling plan for attributes is as follows:

Designing of this sampling plan needs determination of two parameters namely n and c based on some given conditions. These plans are usually denoted by (n, c) where n denotes the sample size and c denotes the acceptance number. When these parameters are known, the sampling plan can be implemented. A single sample of n items is selected at random from a lot of size N and the rejection of the lot is determined from the resulting information. Let d be the observed number of defectives in the sample. If  $d \le c$  then the lot will be accepted otherwise it is rejected.

Many researchers including Guenther (1969), Schilling (1982), Duncan (1986) and Montgomery (2005) have made wide and significant contributions on designing single sampling plans for attributes. The expressions for n and c are obtained by making use of two points on the OC curve of the plan namely,  $(p_1, 1-\alpha)$  and  $(p_2, \beta)$  where  $P_1$  is Acceptance Quality Level (AQL) and  $P_2$  is Limiting Quality Level (LQL) with specified producer's risk  $\alpha$  and the consumer's risk  $\beta$ . Singh and Palanki (1976) found an expression for the lot quality levels and discussed the methods for determining the parameters of binomial single sampling plan using the quality levels. Many researchers including Schilling (1982) discussed the procedure for determining the parameters based on the operating ratio and unit values when the Poisson approximation is appropriate.

#### **Single Sampling Plans for Variables**

A brief description of single sampling plan for variables as described in Duncan (1986) when the observations measured are assumed to be normally distributed with known standard deviation  $\sigma$  is given below.

Let the quality characteristic X be measurable under continuous scale with a probability distribution  $N(\mu, \sigma)$  and L be the specification limit so that an item is declared as defective if it fails to meet the specification namely, X < L or X > U where U be the upper specification limit.

The operating procedure of the single sampling plan for variables is defined as follows: Let  $x_1, x_2, ..., x_n$  be a random sample of n units drawn from a lot submitted for inspection. When  $\sigma$  is known, the lot is accepted if  $\overline{x} + k\sigma \ge L$ otherwise the lot is rejected. As in the case of single sampling plan for attributes, the sampling plan for variables is designated by two parameters n (sample size) and k (acceptance criteria).

#### **b.** Double Sampling Plan

The double sampling plan is designed to give another chance to a doubtful lot. In double sampling plan, if the results of the first sample are not convincing regarding accepting or rejecting the lot then a second sample is taken. Double sampling plan is designated by four parameters namely,

- $n_1$  the size of the first sample
- $n_2$  the size of the second sample
- $c_1$  acceptance number of the first sample
- $\boldsymbol{c}_2$  acceptance number for both samples

The operating procedure of the plan is as follows: Count the number of defectives from the first sample of size  $n_1$  and denote it as  $d_1$ . If  $d_1 \le c_1$  then the lot is accepted based on the first sample and if  $d_1 > c_2$  the lot is rejected based on the first sample. If  $c_1 < d_1 \le c_2$  then the second sample of size  $n_2$  is drawn and number of defectives  $d_2$  in this sample are observed. If  $d_1 + d_2 \le c_2$  then the lot is accepted otherwise it is rejected. The parameters required for constructing OC curve are same as that of the single sampling plan. The two points on the OC curve of the plan namely,  $(p_1, l-\alpha)$  and  $(p_2, \beta)$  where  $p_1$  is the acceptance quality level and  $p_2$  the limiting quality level for a plan with specified producer's risk  $\alpha$  and the consumer's risk  $\beta$ . As far as sample sizes are concerned, the second sample size must be equal to or a multiple of the first sample size.

As an extension of the single sampling plan, Dodge and Romig (1959) have considered double sampling plan. A trial and error procedure for designing double sampling plan for a specified consumer's risk and producer's risk has been developed by Guenther (1970). Schilling and Johnson (1980) have constructed tables for the evaluation of matched set of the double sampling plan. Soundararajan and Arumainayagam (1990) have provided tables for easy selection of double sampling plan indexed through AQL and LQL. For detailed information about double sampling plan refer Duncan (1986) and Schilling and Neubauer (2008).

#### c. Multiple Sampling Plan

This is an extension of the double sampling plan where more than two samples are needed to reach a conclusion. The advantage of multiple sampling is that it has smaller sample sizes but more complex to implement. Multiple sampling plans inspect a maximum of k successive samples from a specified lot to make the decision about it. In MIL-STD 105E (1989), where the value of k is predetermined. It is usually taken as 7.

The operating procedure for multiple sampling plan is as follows: A sample of size  $n_1$  is drawn from a lot of size N and the number of defectives  $d_1$  in the sample are counted. If  $d_1 \le a_1$  then the lot is accepted and if  $d_1 \ge r_1$  then the lot is rejected where  $a_1$  and  $r_1$  are the respective acceptance number and rejection number based on the first sample. If  $a_1 < d_1 < r_1$  then another sample is taken from the lot. The procedure used in the first sample is repeated sample by sample if subsequent samples are needed. The total number of defectives found in the  $i^{th}$  stage defined by  $D_i = \sum_{j=1}^i d_j$  is compared with the acceptance number  $a_i$  and rejection number  $r_i$  for that stage. A decision regarding acceptance or rejection must be made before the number of samples inspected exceeds k. Duncan (1986), Schilling and Johnson (1980) have constructed tables for multiple sampling plans.

#### d. Sequential Sampling Plan

This is the ultimate extension of multiple sampling when there is no predetermined number of samples to be chosen is fixed. In this sampling plan, a sequence of samples is taken from the lot and the number of samples is determined by the results of the sampling process. If the sample size selected for each sequence of the process is greater than one then the process is called group sequential sampling plan. If the sample size selected at each stage is one then the sampling process is called item by item sequential sampling plan. If the sample size selected at each stage is one then the sampling process is called item by item sequential sampling plan. Item by item sequential sampling plan is based on the sequential probability ratio test (SPRT) developed by Wald (1947). Each time an item is inspected and a decision is made to reject the lot, accept the lot, or continue sampling based on the cumulative results. The observed number of defectives and the total number of items selected up to that time are plotted along the ordinate and the abscissa, respectively. If the plotted points lie within the boundaries of acceptance and rejection lines, another sample is drawn. If the point lies on or above the rejection line then the lot is rejected. If it lies below the acceptance line then the lot is accepted.

#### e. Skip Lot Sampling Plan

Skip lot sampling plans inspect only a fraction of the submitted lots. This type of sampling plans is one of the cost-saving plans in terms of time and effort. This plan should only be used when it has been established that the quality of the submitted product is very good. Dodge (1955) introduced the concept of skip-lot sampling by applying the principle of a continuous sampling plan of type CSP-1 to a series of lots. This plan is labelled as the SkSP-1 and is applicable for bulk

products produced in successive lots. Skip lot inspection plan uses a specified lot inspection plan called "the reference sampling plan".

A skip lot sampling plan is executed as follows: A lot-by-lot inspection is carried out using the reference plan. When a pre-specified number, i of consecutive lots are accepted then skipping inspection is carried out in which a fraction f of the lots is considered. The selection of the members of that fraction is done at random. When a lot is rejected then normal inspection is carried out.

In a skip-lot sampling plan, the parameters are important for calculating the probability of acceptance. Dodge and Perry (1971) extended the concept of skip lot sampling procedure to develop the plan of the type SkSP-2. This skip-lot sampling plan is used in the situation where each lot to be inspected is sampled according to lot inspection plan, called the reference plan. A modified skip-lot sampling plan designated as the MSkSP-1 plan was developed by Parker and Kessler (1981). Schneider and Wilrich (1981) have conducted a simulation study for evaluating the efficiency of the switching rules of the MIL-STD 105D (1963) and the skip-lot sampling system. Carr (1982) developed a new type of skip-lot sampling plan designated as CSP-MSkSP.

#### f. Continuous Sampling Plan

Most of the sampling plans in statistical quality control are developed for lot-by-lot production and inspection. Dodge (1943) invented the concept of continuous sampling planning called as CSP - 1 which is used where product flow is continuous. CSP - 1 is designated by two parameters namely, the sampling fraction f and the clearing number i. Initially all units are inspected 100%. As soon as i consecutive units of product are found to be defect free, 100% inspection is stopped or discontinued and only a fraction f of the units are inspected. These sample units are selected one at a time randomly from the flow of production. In brief, we can say that continuous sampling plan can be carried out in 3 steps.

• inspect all *i* units

- if no defects are found, randomly inspect a sample fraction *f* of units and check again for defects
- whenever a defect is found, correct the defect and repeat step 1.

Dodge and Torrey (1951) developed other continuous sampling plans which include CSP - 2 and CSP - 3. Lieberman and Solomon (1955) developed multi-level CSP-M. Among all these continuous sampling plans, CSP - 1 plan is most widely used.

#### g. Chain Sampling Plan

There are situations in which testing is destructive and very costly. In such situations, sampling plans with small sample size are usually preferred. In such cases, practitioners often tend to choose single sampling plan with smaller sample size and acceptance number zero. A sampling plan of this type may reject the lot even if it has single defective item. To overcome this problem, Dodge (1955) designed an alternative procedure called chain sampling plan and denoted it by ChSP - 1. These sampling plans are useful for the situations where smaller sample sizes are required due to physical as well as economic difficulty in obtaining a sample. To apply the chain sampling plan, lots should be produced under same conditions and are expected to have the same quality.

The operating procedure for ChSP-1 is as follows: From each lot, a sample of size n is selected and the number of defectives is observed. Accept the lot, if the sample has zero defective and reject the lot, if the sample has two or more defectives. If the sample has one defective then the decision regarding the acceptance of the lot is based on the i number of consecutive lots having no defectives preceding the current lot where i is a pre-specified positive integer.

The parameters n and i completely determine the sampling plan ChSP – 1. Clark (1960) provided a discussion on modification and some applications on the OC curve of ChSP – 1 with different combinations of n and i. Soundararajan (1978) constructed tables for the selection of chain sampling plan for the given producer's risk  $\alpha$ , consumer's risk  $\beta$ , acceptable quality level (AQL) denoted as  $p_1$ , and limiting quality level (LQL) denoted as  $p_2$ . Dodge and Stephens (1966) designed a chain sampling plan denoted by ChSP (0-1) which is the generalization of ChSP – 1.

#### h. Tightened Normal Tightened Plan

In a compliance testing, if a single unit in the sample is defective we reject the entire lot. To overcome this unfavorable situation, Tightened Normal Tightened (TNT) scheme was devised by Calvin (1977). This TNT scheme operates two single sampling plans (with c = 0) of different sample sizes together with switching rules. The TNT scheme is designated by

- $n_1$  Tightened normal sample size (larger sample size)
- $n_2$  Normal sample size (smaller sample size)
- t Criterion for switching to normal inspection
- s Criterion for switching to tightened inspection.

This TNT scheme is designated as TNT  $(n_1, n_2; 0)$ . The operating procedure of the scheme is, inspect using tightened inspection with larger sample size  $n_1$  and acceptance number c = 0. Switch to normal inspection when t lots in a row are accepted under tightened inspection. Inspect using normal inspection with smaller sample size  $n_2$  ( $< n_1$ ) and acceptance number c = 0. Switch to tightened inspection after a rejection, if an additional lot is rejected in the next s number of lots.

Thus, the TNT  $(n_1, n_2; c)$  sampling scheme is completely specified by the parameters  $n_1$ ,  $n_2$ , c and s, t the criteria for switching to tightened and normal inspections, respectively.

The expression for composite OC function of the scheme has been derived by Dodge (1965) and Hald and Thyregod (1965). Schilling (1982) has derived an approximate procedure for determining  $n_1$  and  $n_2$  for the prescribed values of  $(p_1, I - \alpha)$  and  $(p_2, \beta)$ . Soundararajan and Vijayaraghavan (1990) constructed a table of parametric values for various TNT scheme. Soundararajan and Vijayaraghavan (1992) studied the TNT scheme by assuming 'c' takes values other than zero and the scheme was designated as TNT  $(n_1, n_2; c)$ . They have also shown that the TNT scheme is efficient over conventional single and double sampling plans. Vijayaraghavan and Soundararajan (1996) designed procedures for the selection of TNT  $(n; c_1, c_2)$  indexed by (AQL, LQL) and (AQL, AOQL) under the application of Poisson model.

#### i. Time Truncated Acceptance Sampling Plan

Acceptance sampling plans based on truncated life tests for exponential distribution was first discussed by Epstein (1954) and also can be seen in Sobel and Tischendrof (1959). The results were extended for the Weibull distribution by Goode and Kao (1961). Gupta and Groll (1961), Gupta (1962), Baklizi (2003), Baklizi and El Masri (2004) and Wenhao Gui and Shangli Zhang (2014) provided time truncated acceptance sampling plans for gamma, normal and log-normal distributions, Pareto distribution of second kind, Birnbaum Saunders model, Gompertz distribution respectively.

The procedure for designing time truncated acceptance sampling plan involves the inspection of *n* items selected from the lot over a given period, say *t*. The lot is accepted if the number of observed failures till time point *t* does not exceed a pre-specified acceptance number *c*. The lot is rejected with the termination of the test if the number of failures observed before the time *t* exceeds the acceptance number *c*. The inspection time *t* is a pre-specified quantity. It is usually taken as a multiple of targeted mean/median life time of the product, say  $\theta_m^0$ . That is, we take  $t = a\theta_m^0$  where *a* is also a pre-determined quantity which indicates the number of cycles needed to guarantee specified mean/median life time of the product. The parameters of the time truncated acceptance sampling plan are:

• the number of items *n* to be drawn from the lot

- an acceptance number c
- the time ratio  $\frac{t}{\theta_m^0}$  where  $\theta_m^0$  is the specified mean/median life time of a product
- the pre-assigned testing time *n*.

Symbolically, the sampling plan is denoted by the triplet  $(n, c, \frac{t}{\theta_m^0})$ .

Arriving at a decision based on a time truncated acceptance sampling plan is equivalent to taking a decision while testing the null hypothesis  $H_0: \theta_m \ge \theta_m^0$ against the alternative hypothesis  $H_1: \theta_m < \theta_m^0$  at a given level of significance  $1-P^*$  which is nothing but the consumer's risk. Kantam and Rosaiah (1998), Kantam et al. (2001), Rosaiah and Kantam (2001) and Balakrishnan et al. (2007) provided the time truncated acceptance plans for half-logistics, log-logistics, Rayleigh and generalized Birnbaum-Saunders distributions, respectively.

#### j. Reliability Sampling plan

If the quality of a product is assessed using its life time then a randomly selected lot of such products is subjected to life testing process. The decision to accept or reject a lot is based on the risk associated with two types of errors namely, Type-I error (rejecting a good quality lot) and Type-II error (accepting a bad quality lot). Procedures based on such life testing process are termed as 'Reliability test plans' or 'Acceptance sampling plans based on life test'. Designing of such test procedures (sampling plans) requires the identification of an appropriate probability model governing the life time of products. Economic reliability test plans were studied under log-logistic distribution and generalized exponential distribution by Kantam, Srinivasa Rao and Sriram (2006) and Aslam and Shabaz (2007), respectively. Rao, Ghitany and Kantam (2009), Rao, Kantam, Rosaiah and Prasad (2012) provided reliability test plan for Marshal-Olkin

extended Lomax distribution, type-II exponentiated log-logistic distribution, respectively.

The operational procedure of economic reliability test plan is explained below. Let *n* be the size of sample taken for inspection from a lot and *r* be the termination number. The lot is rejected, if *r* failures out of *n* items occur before the termination time  $t_0$  otherwise it is accepted. The experiment is stopped as soon as the termination time  $t_0$  or the  $r^{th}$  failure is reached, whichever is earlier.

One of the important factors in life test experiment is the selection of sample size which plays a crucial role in the performance of life test experiment. The sample size is selected in such a way that the expected waiting time to reach the decision and cost of the experiment are optimized. Kantam et al. (2006) considered the sample size as a multiple of the termination number r. The probability of accepting the lot is given by  $\sum_{i=0}^{r-1} {n \choose i} p^i q^{n-i}$ . Kantam et al. (2006) determined the value of p in reliability test plan by using the cumulative distribution function of binomial distribution.

#### k. Repetitive Group Sampling Plan

Sherman (1965) designed a Repetitive Group Sampling (RGS) plan which is used for making decisions about isolated lots of finished products. This plan is designated by three parameters, they are

- i) n, the sample size
- ii)  $c_1$ , the first acceptance number
- iii)  $c_2$ , the second acceptance number.

The RGS plan is denoted by RGS (n;  $c_1$ ,  $c_2$ ). The operating procedure is as follows.

A sample of size *n* is selected from the given lot and the number of defectives *d* is observed. The lot is accepted, if  $d \le c_1$ , rejected, if  $d > c_2$ . If  $c_1 < d \le c_2$  then repeat the procedure from the beginning and continue the process until the lot is either rejected or accepted.

Soundararajan and Ramasamy (1984) developed procedures for designing RGS plans

- i) when two points on the OC curve namely,  $(p_1, 1-\alpha)$  and  $(p_2, \beta)$  are prescribed.
- ii) when  $(p_{1,}1-\alpha)$  on the OC curve and average outgoing quality level (AOQL) are given as requirements.

Singh, Shankar and Mohapatra (1989) evaluated the OC function of RGS plan through graphical evaluation and review technique. As an extension of classical RGS plan, Mohapatra (1992), Shankar and Mohapatra (1993) developed a conditional RGS plan. The concept of two phase RGS inspection using graphical evaluation and review technique for measuring the performance of OC and average sample number (ASN) was introduced by Shankar (1996).

# **1.3 Measures of Sampling Plan**

This section provides the measures which are required to design a sampling plan.

- 1. Acceptable Quality Level (AQL): The poorest level of quality for the supplier's process that the consumer would consider to be acceptable as a process average. The AQL is a percent defective that is the minimum requirement for the quality of the producer's product. Sampling plan should be designed with high probability of accepting a lot that has a defective level less than or equal to the AQL. The probability of acceptance of a lot with a process average equal to the AQL is normally set at 0.95.
- 2. Lot Tolerance Percent Defective (LTPD): It is a level of quality routinely rejected by the sampling plan. It is the poorest level of quality (percent defective) that the consumer willing to accept in an individual lot. The LTPD is not a characteristic of the sampling plan, but is a level of quality specified by the consumer. Alternate names for LTPD are the rejectable quality level and the limiting quality level.
- 3. **Type I Error (Producer's Risk)**: Rejecting the null hypothesis when it is true is called type I error. The probability of committing type I error is denoted by  $\alpha$ . It is the level of significance for hypothesis testing. The

producer suffers when this error occurs, because a lot with acceptable quality is rejected.

- 4. **Type II Error (Consumer's Risk)**: Accepting the null hypothesis when it is false is called type II error. The probability of committing type II error is denoted by  $\beta$ . This is nothing but the probability of accepting a lot with a defective level equal to the LTPD. The consumer suffers when this error occurs, because a lot with unacceptable quality is accepted.
- 5. Average Outgoing Quality (AOQ): It is mainly used for the evaluation of a rectifying sampling plan. It is the average value of lot quality that would be obtained over a long sequence of lots from a process with fraction defective p. The average quality level of an outgoing product for a given value of incoming product quality.
- 6. Average Outgoing Quality Level (AOQL): The maximum ordinate on the AOQ curve represents the worst possible average quality that would result from the rectifying inspection program, and this point is called AOQL.
- 7. Average Total Inspection (ATI): The average number of units inspected per lot including all units in rejected lots.
- 8. Average Sample Number (ASN): The average number of sample units examined per lot in reaching decision to accept or reject a lot for a given sampling plan. ASN curve for a sampling plan is obtained by plotting ASN against proportion of defectives *p* which describes the sampling efficiency of a given sampling plan.
- 9. **Probability of Acceptance**  $(P_a)$ : The probability value for a lot being accepted for a given sampling plan.
- 10. **Operating Characteristic (OC) Curve**: This curve is the primary tool for exhibiting and examining the properties of acceptance sampling plan. OC curve plots the probability of accepting the lot  $P_a$  along *y*-axis versus the percent defectives *p* along *x*-axis. The OC curve measures the performance of an acceptance-sampling plan. It shows the probability that a lot submitted with a certain fraction defective will be either accepted or rejected.

OC curves are classified into two types as per ANSI/ASQC standard (1987), they are i) Type A OC curve and ii) Type B OC curve. Type A OC curve is the most suitable one when samples are drawn from a unique lot or isolated lot or a lot from the isolated sequence. The probability of accepting a lot is defined as a function of lot proportion defective for a given acceptance sampling plan as used under Type A OC curve. This OC curve represents the probability of accepting a lot as a function of process proportion defective for a given sampling plan. Type B OC curve is for continuous stream of lots.

# 1.4 Conclusion

In this introductory chapter, some fundamental concepts of statistical quality control and statistical measures associated with the evaluation of acceptance sampling plans are explained. A complete review of different types of acceptance sampling plans and their operating procedures has been carried out.