DSE III, Sixth semester

UNIT II
Special theory of Relativity (1905)

Books! - 1. Introduction to Special Relativity
Robert Resnick

2. Concept of Modern Physics
- Arther Beiser

3. Classical Mechanics -
- Gupta Kumar
- Takwale & Puranik

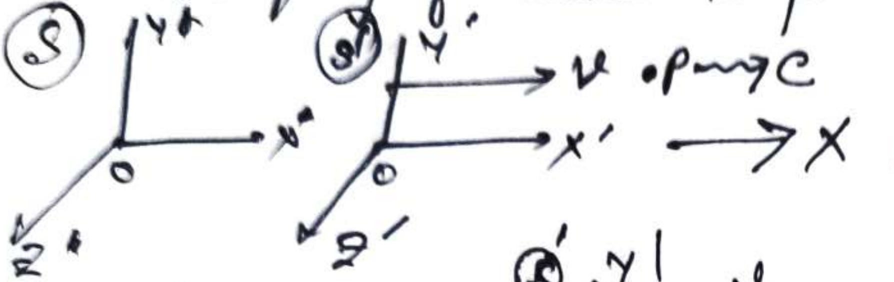
Basic postulates of Special theory of Relativity

1. The laws of physics are the same in all inertial frame of reference.
In other words no preferred inertial systems exists (Principle of Relativity)

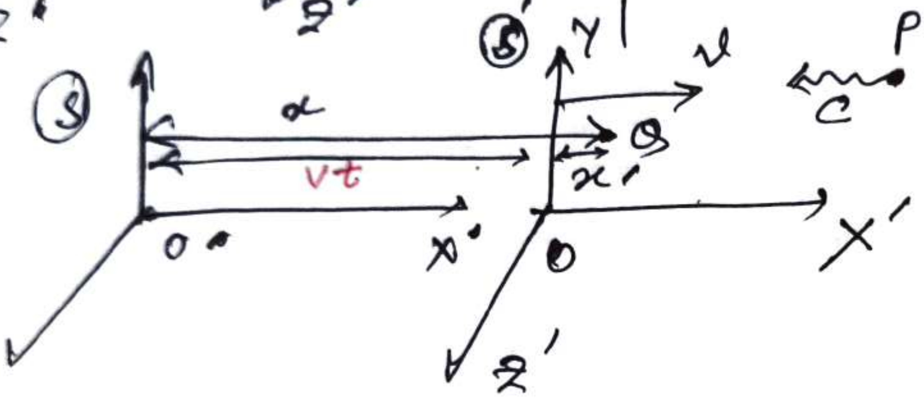
2. Light travels in vacuum with a velocity c in all inertial systems. (Principle of Constancy of the velocity of light)

- What is transformation ?
- Previous Concept of transformation :- Galilean transformation

- Inadequacy of Galilean transformation



$\equiv c + v$



$= c - v$

$$\left.\begin{aligned} x' &= x - vt \\ y' &= y \\ z' &= z \\ t' &= t \end{aligned}\right\}$$

Controversial Result w.r.to . STR.

Need Some correction ?
what will be the possible correction?

Lorentz put forward a New set of transformation equation. Popularly known as Lorentz Transformation !

At $t = t' = 0$

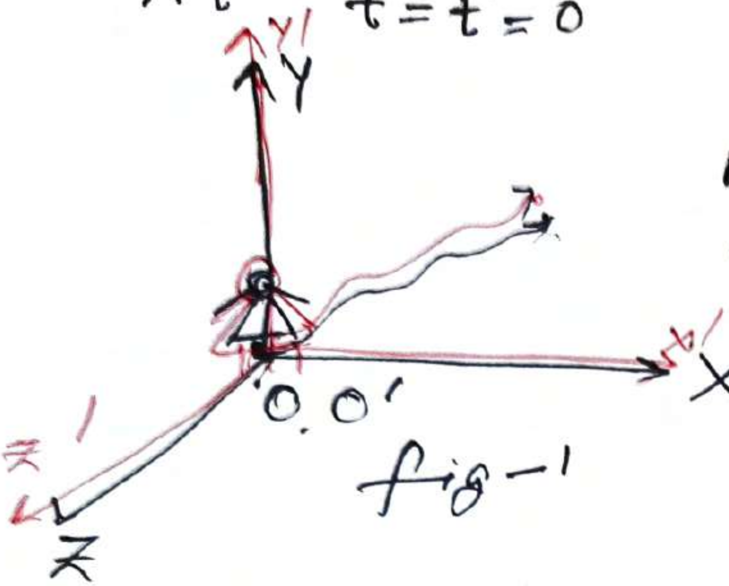


fig-1

Both the observers are ~~at~~ at the same point and both of them are at the centre of the expanding wavefront.

~~After t~~ After some instant of time t

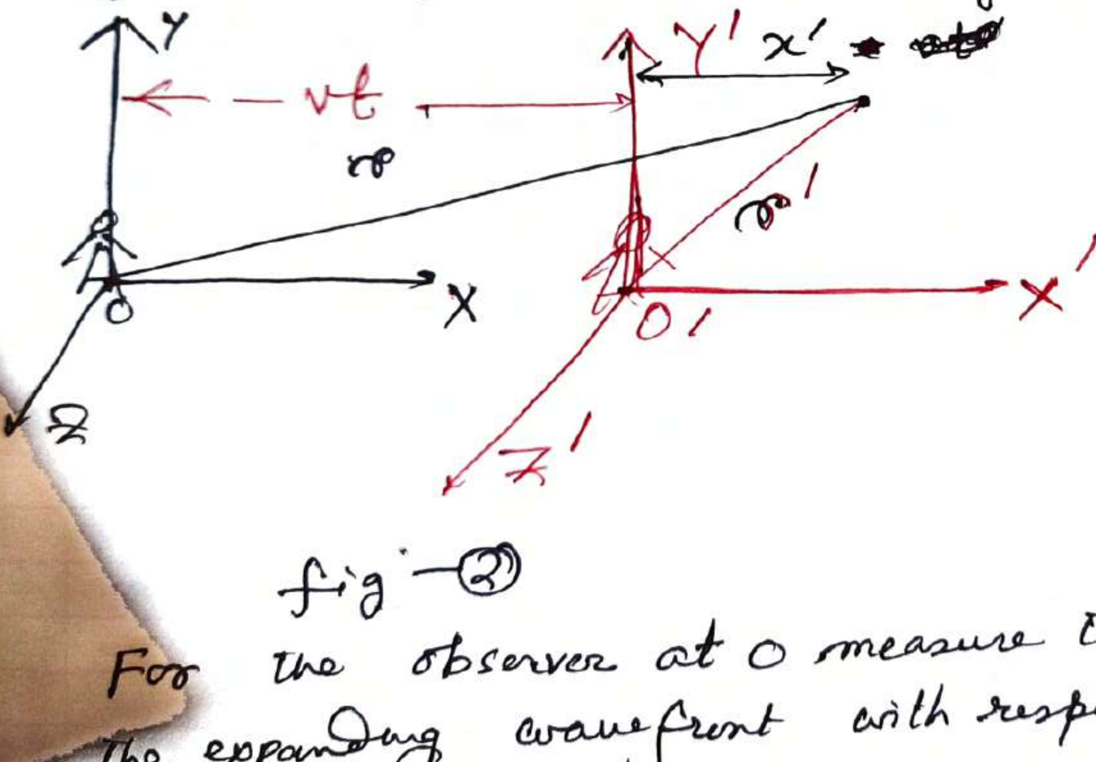


fig-(2)

For the observer at O measure the radius of the expanding wavefront with respect to her own co-ordinate system.

$$\therefore\ r^2 = x^2 + y^2 + z^2 \longrightarrow (1)$$

The observer at O' measures the radius of the expanding wavefront with respect to her co-ordinate system -

$$r'^2 = x'^2 + y'^2 + z'^2 \longrightarrow (2)$$

Equation (1) and (2) give after rearrangement & replacement of $r = ct$ and $r' = ct'$

$$x^2 + y^2 + z^2 - c^2t^2 = x'^2 + y'^2 + z'^2 - c^2t'^2 \longrightarrow (3)$$

According to Galilean transformation the relativistic effect is experienciable only along the direction of relative motion so

$$y = y', \quad z = z'$$

$\therefore$ equation (3) gives

$$x^2 - c^2t^2 = x'^2 - c^2t'^2 \longrightarrow (4)$$

After testing different order of space and time coordinate dependence of one another it was found that the most ~~suitable and~~ reliable relation between x and x' and t and t' are linear. Let the relations

[Resnick page 57 2005]

are as follows

$$x' = k(x - vt) \longrightarrow (5a) \quad \text{and} \quad t' = a(t - bx) \rightarrow (5b)$$

using the relations in equation (4)

$$x^2 - c^2t^2 = k^2(x^2 + v^2t^2 - 2xvt) - c^2a^2(t^2 + b^2x^2 - 2xbt)$$

$$\Rightarrow x^2 - c^2t^2 = (k^2 - c^2a^2b^2)x^2 - 2xt(k^2v - c^2a^2b) - \left(a^2 - \frac{k^2v^2}{c^2}\right)c^2t^2$$

Equating the coefficient of x and vt in the both sides of the equations:

$$\left.\begin{aligned} k^2 - c^2a^2b^2 &= 1 \quad (6a)\\ k^2v - c^2a^2b &= 0 \quad (6b)\\ a^2 - \frac{k^2v^2}{c^2} &= 1 \quad (6c) \end{aligned}\right\}$$

The three variables associated with the 3 equations are k, a and b

Solving gives

$$\left.\begin{aligned} k = a &= \frac{1}{\sqrt{1 - v^2/c^2}} \\ b &= \frac{v}{c^2} \end{aligned}\right\} (7)$$

using equation (7) in (5a) and (5b) $\Rightarrow$

$$x' = \frac{x - vt}{\sqrt{1 - v^2/c^2}} \quad \& \quad t' = \frac{t - vx/c^2}{\sqrt{1 - v^2/c^2}}$$

Now for the four sets of axis of the two co-ordinate system the transformation relations become

$$\left.\begin{aligned} x' &= \frac{x - vt}{\sqrt{1 - v^2/c^2}} \\ y' &= y \\ z' &= z \\ t' &= \frac{t - vx/c^2}{\sqrt{1 - \frac{v^2}{c^2}}} \end{aligned}\right\} \quad (8)$$

In some books $\frac{v}{c} = \beta$ and $\frac{1}{\sqrt{1 - v^2/c^2}} = \gamma$

$\therefore$ using the above notations the equation (8) becomes

$$\left.\begin{aligned} x' &= \gamma(x - vt) \\ y' &= y \\ z' &= z \\ t' &= \gamma(t - vx/c^2) \end{aligned}\right\} (9) \quad \text{or} \quad \left.\begin{aligned} x' &= \frac{x - vt}{\sqrt{1 - \beta^2}} \\ y' &= y \\ z' &= z \\ t' &= \frac{t - \beta^2 \frac{x}{v}}{\sqrt{1 - \beta^2}} \end{aligned}\right\} (10)$$

Equⁿ (8), (9) and (10) the 3 sets of equations are known as direct Lorentz transformation.

[In direct Lorentz transformation the R.H.S. quantities are known and they are associated with the frame which is rest with respect to the observer]

In the reverse case ie if the ~~measurement~~ frame of reference is exchanged. The transformation relation then takes the form

$$\left.\begin{aligned} x &= \frac{x' + vt'}{\sqrt{1 - v^2/c^2}} \\ y &= y' \\ z &= z' \\ t &= \frac{t' + \frac{vx'}{c^2}}{\sqrt{1 - v^2/c^2}} \end{aligned}\right\} \quad (11)$$

This set of equation is known as inverse Lorentz transformation.

** [Lorentz in his calculation consider one frame is attached to the ether frame of reference and at absolute rest. Einstein performed the relative calculations]

* For small speed compared to c ie $V/c << 1$, Lorentz tranformation reduce to Galilean tranformation.

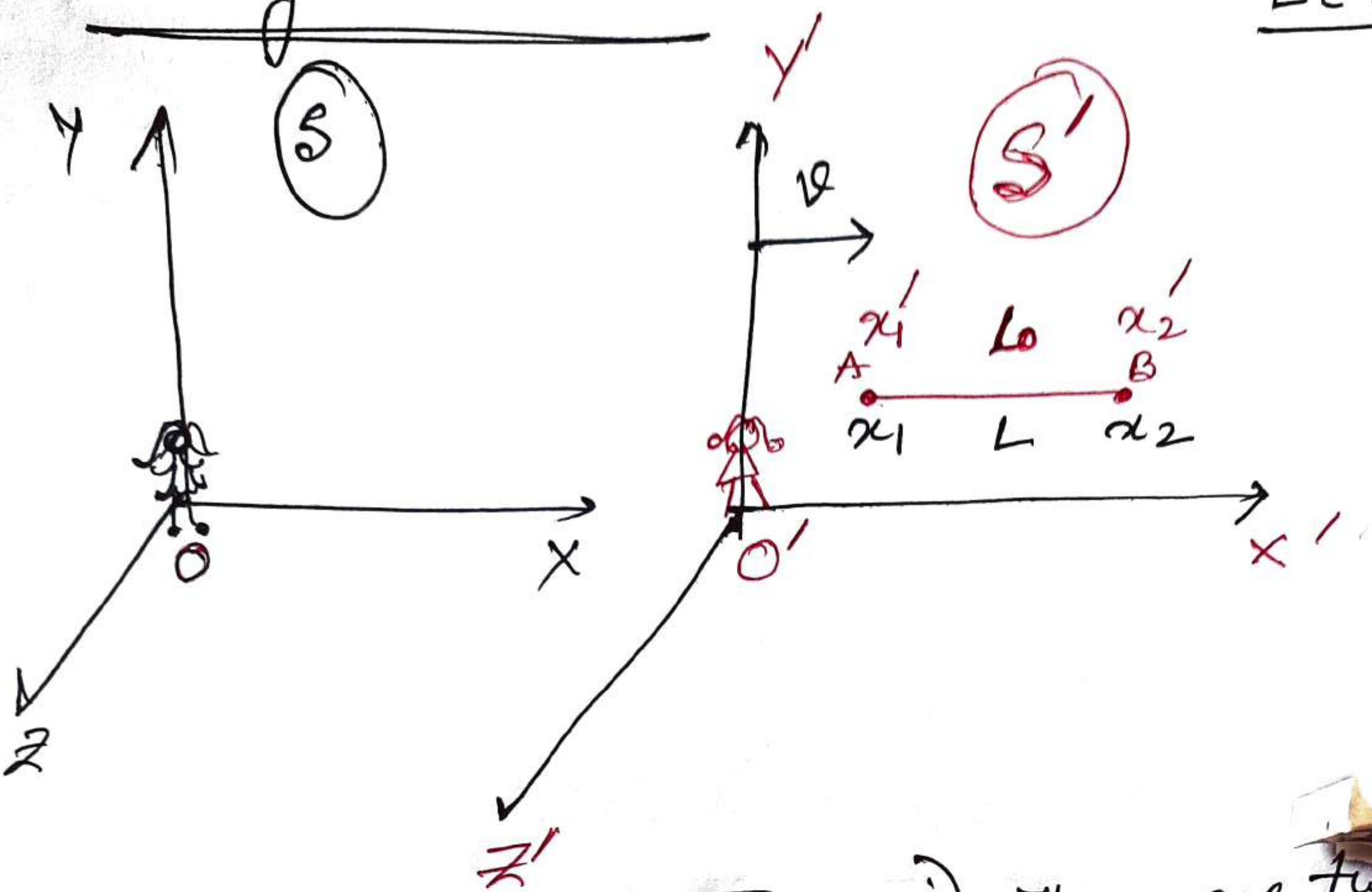
Consequences of Lorentz transformation.

(i) Length Contraction

(2) Time dilation (• Time interval Measurement
• Simulteneity)

(3) Meson decay [Practical or Natural example of Lorentz transformation]

Length Contraction



Description of the Systems : i) There are two frames of reference S and S'.

(ii) The frame S is at rest (Rest with respect to us) and the frame S' is moving with a constant translational velocity v along the positive X ~~X'~~ direction.

(iii) Let a rod is placed along the X' axis of the S' ~~fr~~ frame. Let the length of the rod measured by an observer at ~~O~~ O' is L_0.

(iv) Let the same rod is measured by another observer at O is L.

We are to set a relation betn the measurements of the two observer.

L_0 → The length measured by the observer when she or he is at rest with respect to the rod. This length is called the <u>proper length</u>.

L → The length of the rod mesured by the observer when there is a relative motion betn the observer and the object measured. This length is called the <u>relativistic length</u>.

Calculation:

Let x_1' and x_2' are the two ends of the rod AB with respect to the observer at O' at any instant of time.

$$\therefore L_0 = x_2' - x_1' \longrightarrow (1)$$

Let x_1 and x_2 are the same two ends of the rod AB as measured by the observer at O at the same instant of time.

$$\therefore L = x_2 - x_1 \longrightarrow (11)$$

Applying direct Lorentz transformation in equation ①

$$L_0 = \frac{x_2 - vt}{\sqrt{1-\frac{v^2}{c^2}}} - \frac{x_1 - vt}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\Rightarrow L_0 = \frac{x_2 - x_1}{\sqrt{1-v^2/c^2}}$$

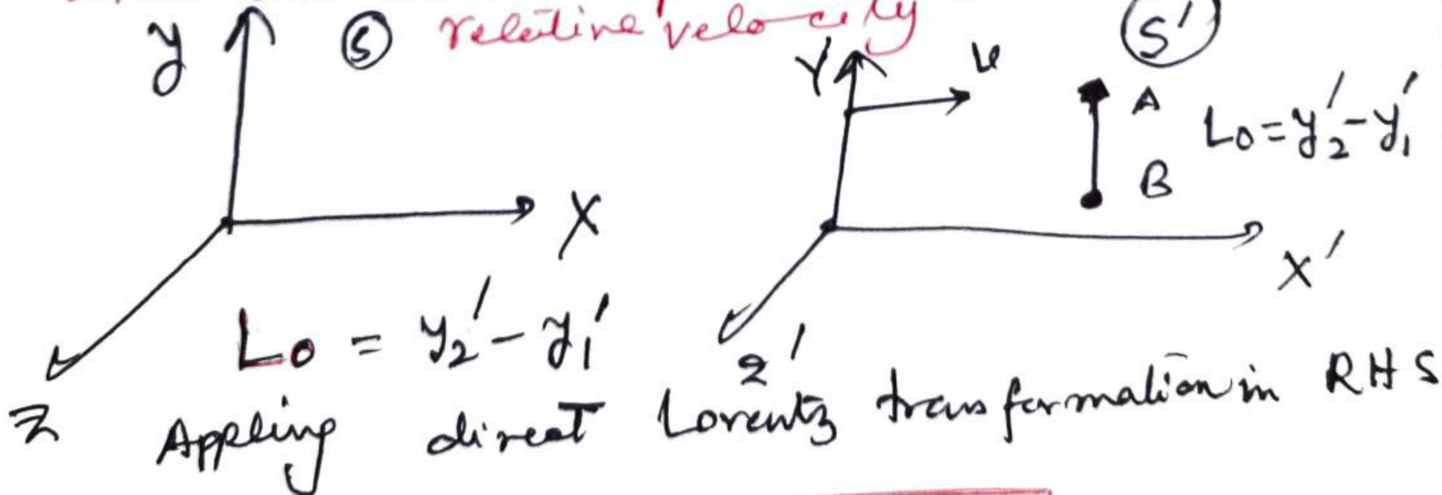
$$\Rightarrow L_0 = \frac{L}{\sqrt{1-v^2/c^2}}$$

$$\therefore L = L_0\sqrt{1-v^2/c^2}$$

As $\sqrt{1-v^2/c^2} < 1$; $L < L_0$

$\therefore$ Proper length is always ~~the longest~~ longer then the relativistic length.

- When the rod is placed perpendicular to the relative velocity



$L_0 = y_2' - y_1'$

$L_0 = y_2' - y_1'$

Appling direct Lorentz transformation in RHS

$$L_0 = y_2 - y_1 = L$$

$$\Rightarrow \boxed{L_0 = L}$$