Assignments

Attempt all Questions.

Example 18.5. Let $X_1, X_2, ..., X_{20}$ be a random sample from a distribution with probability density function

$$f(x;p) = \begin{cases} p^x(1-p)^{1-x} & \text{if } x = 0, 1 \\ \\ 0 & \text{otherwise,} \end{cases}$$

where $0 < p \leq \frac{1}{2}$ is a parameter. The hypothesis $H_o : p = \frac{1}{2}$ to be tested against $H_a : p < \frac{1}{2}$. If H_o is rejected when $\sum_{i=1}^{20} X_i \leq 6$, then what is the probability of type I error?

Example 18.6. Let p represent the proportion of defectives in a manufacturing process. To test $H_o : p \leq \frac{1}{4}$ versus $H_a : p > \frac{1}{4}$, a random sample of size 5 is taken from the process. If the number of defectives is 4 or more, the null hypothesis is rejected. What is the probability of rejecting H_o if $p = \frac{1}{5}$?

Example 18.7. A random sample of size 4 is taken from a normal distribution with unknown mean μ and variance $\sigma^2 > 0$. To test $H_o : \mu = 0$ against $H_a : \mu < 0$ the following test is used: "Reject H_o if and only if $X_1 + X_2 + X_3 + X_4 < -20$." Find the value of σ so that the significance level of this test will be closed to 0.14.

Example 18.8. A normal population has a standard deviation of 16. The critical region for testing $H_o : \mu = 5$ versus the alternative $H_a : \mu = k$ is $\bar{X} > k - 2$. What would be the value of the constant k and the sample size n which would allow the probability of Type I error to be 0.0228 and the probability of Type II error to be 0.1587.

Example 18.9. A manufacturing firm needs to test the null hypothesis H_o that the probability p of a defective item is 0.1 or less, against the alternative hypothesis $H_a : p > 0.1$. The procedure is to select two items at random. If both are defective, H_o is rejected; otherwise, a third is selected. If the third item is defective H_o is rejected. If all other cases, H_o is accepted, what is the power of the test in terms of p (if H_o is true)?

Example 18.10. Let X be the number of independent trails required to obtain a success where p is the probability of success on each trial. The hypothesis $H_o : p = 0.1$ is to be tested against the alternative $H_a : p = 0.3$. The hypothesis is rejected if $X \leq 4$. What is the power of the test if H_a is true?

Example 18.11. Let $X_1, X_2, ..., X_{25}$ be a random sample of size 25 drawn from a normal distribution with unknown mean μ and variance $\sigma^2 = 100$. It is desired to test the null hypothesis $\mu = 4$ against the alternative $\mu = 6$. What is the power at $\mu = 6$ of the test with rejection rule: reject $\mu = 4$ if $\sum_{i=1}^{25} X_i \geq 125$?

Example 18.12. A urn contains 7 balls, θ of which are red. A sample of size 2 is drawn without replacement to test $H_o : \theta \leq 1$ against $H_a : \theta > 1$. If the null hypothesis is rejected if one or more red balls are drawn, find the power of the test when $\theta = 2$.

Problem 1: What are the points which need special attention at the planning stages of sample surveys? With reference to any suitable example (preferably surveys actually conducted in India), explain whether these have been covered or not.

Problem 2: A population contains N units, the variate value of one unit being known to be y_0. A random sample, wor, is drawn from the remaining (N - 1) units. Show that the estimator $y_0 + (N - 1)\bar{y}$ has a smaller variance than $N\bar{y}$ based on a random sample, wor, of size n taken from the whole population.

Problem 3: Derive the expression for variance of a simple random sample drawn without replacement from a finite population.

N balls, placed in a lot container, are drawn at random from a supply of M_p red and M_q white balls. Then a sample of n balls is drawn at random from the lot container and placed in a sample container. It is found that, out of these n balls, r are red. Find the Var (r) when the N balls are put into the lot container (i) with replacement, and (ii) without replacement.

Problem 4: Obtain the expression of expectation and variance of the sample mean drawn from a finite population of size N with unknown mean μ, and unknown variance θ^2, when sampling is (i) with replacement, and (ii) without replacement.

Problem 5: Draw a random sample of size 10 from a normal population $N(50, 10^2)$.